i.e. $Q_{12} = 2.347 \times 10^4$ **Watt.**

Additionally:

If both the surface are black Now, both the surface resistances become zero, since $\varepsilon = 1$ for both the black surfaces. Then,

$$Q_{12} := \frac{E_{b1} - E_{b2}}{R_{12}}$$

i.e. $Q_{12} = 4.988 \times 10^4$ W *(if both the surfaces are black.)*

Example 13.19. Refer to Fig. Example 13.19. Three thin-walled, long, circular cylinders 1, 2 and 3, of diameters 15 cm, 25 cm and 35 cm, respectively, are arranged concentrically as shown. Temperature of cylinder 1 is 80 K and that of cylinder 3 is 300 K. Emissivities of cylinders 1, 2 and 3 are 0.05, 0.1 and 0.2, respectively. Assuming that there is vacuum inside the annular spaces, determine the steady state temperature attained by cylinder 2.

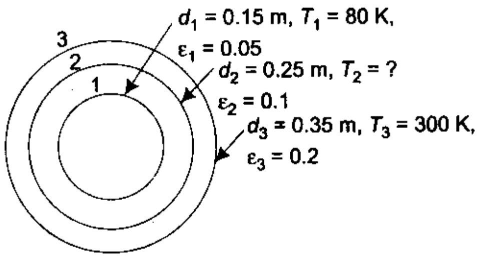


FIGURE Example 13.19 Three concentric cylinders

Solution. This is the case of radiant heat transfer between long, concentric cylinders.

Data:

$T_1 := 80$ K $\quad T_3 := 300$ K $\quad \varepsilon_1 := 0.05 \quad \varepsilon_2 := 0.10 \quad \varepsilon_3 := 0.20 \quad d_1 := 0.15$ m $\quad d_2 := 0.25$ m

$d_3 := 0.35$ m $\quad \sigma := 5.67 \times 10^{-8}$ W/(m²K)

Let L be the length of cylinders. Then,

$$\frac{A_1}{A_2} = \frac{\pi \cdot d_1 \cdot L}{\pi \cdot d_2 \cdot L} = \frac{d_1}{d_2}$$

and,

$$\frac{A_2}{A_3} = \frac{\pi \cdot d_2 \cdot L}{\pi \cdot d_3 \cdot L} = \frac{d_2}{d_3}$$

In steady state, net radiant heat transfer between cylinders 1 and 2 must be equal to the net radiant heat transfer between cylinder 2 and 3.

i.e. $$Q_{12} := Q_{23} \qquad \text{...(A)}$$

We apply Eq. 13.60, namely,

$$Q_{12} = \frac{A_1 \cdot \sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right)}$$

(for infinitely long, concentric cylinders...(13.60))

Applying Eq. 13.60 in heat balance Eq. A:

$$\frac{A_1 \cdot \sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right)} = \frac{A_2 \cdot \sigma \cdot (T_2^4 - T_3^4)}{\frac{1}{\varepsilon_2} + \left(\frac{A_2}{A_3}\right) \cdot \left(\frac{1}{\varepsilon_3} - 1\right)} \qquad \text{...(B)}$$

In the above equation, T_2 is the only unknown. Simplifying Eq. B:

$$\frac{A_1 \cdot (T_1^4 - T_2^4)}{A_2 \cdot (T_2^4 - T_3^4)} = \frac{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right)}{\frac{1}{\varepsilon_2} + \left(\frac{A_2}{A_3}\right) \cdot \left(\frac{1}{\varepsilon_3} - 1\right)}$$

i.e. $$\frac{(T_1^4 - T_2^4)}{(T_2^4 - T_3^4)} = \frac{\frac{1}{\varepsilon_1} + \left(\frac{d_1}{d_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right)}{\frac{1}{\varepsilon_2} + \left(\frac{d_2}{d_3}\right) \cdot \left(\frac{1}{\varepsilon_3} - 1\right)} \cdot \left(\frac{d_2}{d_1}\right)$$

(since $\frac{A_1}{A_2} = \frac{d_1}{d_2}$ and, $\frac{A_2}{A_3} = \frac{d_2}{d_3}$)

i.e. $$\frac{(T_1^4 - T_2^4)}{(T_2^4 - T_3^4)} = 3.293$$

i.e. $$(T_1^4 - T_2^4) = 3.293 \cdot T_2^4 - 3.293 \cdot T_3^4$$

i.e. $$4.293 \cdot T_2^4 = T_1^4 + 3.293 \cdot T_3^4$$

i.e. $$T_2 := \left(\frac{T_1^4 + 3.293 \cdot T_3^4}{4.293}\right)^{\frac{1}{4}}$$

i.e. $T_2 = 280.864$ K *(steady state temperature of cylinder 2.)*

Alternatively:

We can get value of T_2 very easily by applying solve block of Mathcad.

Start with a guess value for T_2, and write the constraint, i.e. Eq. B immediately after 'Given' in the solve block; then, typing 'Find(T_2) =' gives the value of T_2:

$T_2 := 200$ K ...*guess value*)

Given

$$\frac{d_1 \cdot \sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \left(\frac{d_1}{d_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right)} = \frac{d_2 \cdot \sigma \cdot (T_2^4 - T_3^4)}{\frac{1}{\varepsilon_2} + \left(\frac{d_2}{d_3}\right) \cdot \left(\frac{1}{\varepsilon_3} - 1\right)}$$

$$\text{Find}(T_2) = 280.862$$

i.e. $T_2 = 280.862$ **K** *(same as obtained earlier.)*

Note: While writing the constraint equation in the solve block above, we have substituted d_1/d_2 for A_1/A_2,and d_2/d_3 for A_2/A_3.

Once again, it is demonstrated that using solve block of Mathad, very much simplifies the solution, and reduces the labour involved.

Example 13.20. A blind cylindrical hole of diameter 2 cm and length 3 cm is drilled into a metal slab having emissivity 0.6. If the metal slab is maintained at a temperature of 350°C, find the heat escaping out of the hole by radiation. (M.U)

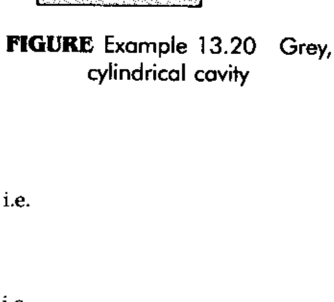


FIGURE Example 13.20 Grey, cylindrical cavity

Solution. This is a problem on determining energy esaping from a grey cavity. We use Eq. 13.62, i.e.

$$Q_{12} = A_1 \cdot \varepsilon_1 \cdot \sigma \cdot T_1^4 \cdot \left[\frac{1 - F_{11}}{1 - (1 - \varepsilon_1) \cdot F_{11}}\right] \text{W}$$

(net radiation from grey cavity)

Data:

$D := 0.02$ m $\quad H := 0.03$ m $\quad \varepsilon_1 := 0.6 \quad T_1 := 350 + 273$ K

$\sigma := 5.67 \times 10^{-8}$ W/(m²K) *(Stefan-Boltzmann constant)*

Now, F_{11} for a cavity is already shown to be:

$F_{11} = 1 - \frac{A_2}{A_1}$ *(where, A_2 = area of closing surface, A_1 = area of the cavity surface)*

i.e. $$F_{11} = 1 - \frac{\frac{\pi \cdot D^2}{4}}{\frac{\pi \cdot D^2}{4} + \pi \cdot D \cdot H}$$ *(for cylindrical cavity of this problem)*

i.e. $$F_{11} = 1 - \frac{D}{D + 4 \cdot H} = \frac{4 \cdot H}{4 \cdot H + D}$$

Therefore, $$F_{11} := \frac{4 \cdot H}{4 \cdot H + D}$$

i.e. $F_{11} = 0.857$ *(view factor of the cavity w.r.t. itself)*

and, $$A_1 := \pi \cdot D \cdot H + \frac{\pi \cdot D^2}{4}$$

i.e. $A_1 = 2.199 \times 10^{-3}\ m^2$ *(area of surface of cylindrical cavity)*

Then, from Eq. 13.62:

$$Q_{12} := A_1 \cdot \varepsilon_1 \cdot \sigma \cdot T_1^4 \cdot \left[\frac{1 - F_{11}}{1 - (1 - \varepsilon_1) \cdot F_{11}}\right]$$

i.e. $Q_{12} = 1.063\ \mathbf{W}$ ***(energy escaping from the cavity.)***

Example 13.21. A hohlraum is to be constructed out of a thin copper sphere of diameter = 15 cm. Its internal surface is highly oxidised. What should be the area of a small opening to be made on the surface of the sphere, if the desired absorptivity is 0.95?

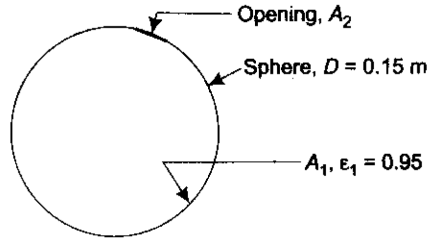


FIGURE Example 13.21 Hole on the surface of a sphere—holhraum

Solution.

Data:

$D := 0.15$ m *(diameter of the sphere)*

$\alpha_1 := 0.95$ *(absorptivity)*

$\varepsilon_1 := \alpha_1$ *(emissivity (= absorptivity), by Kirchhoff's law)*

The inside surface of the sphere must absorb 95% of the energy, which means that 5% of the energy escapes out through the opening of area = A_2, say.

Let Q_2 = energy escaping through the hole, and

Q_1 = energy radiated from the spherical cavity

Then, we have:

$$Q_2 = A_1 \cdot \varepsilon_1 \cdot \sigma \cdot T_1^4 \cdot \left[\frac{1 - F_{11}}{1 - (1 - \varepsilon_1) \cdot F_{11}}\right] \quad \textit{(from Eq. 13.62)}$$

and,

$$Q_1 = A_1 \cdot \varepsilon_1 \cdot \sigma \cdot T_1^4$$

By data,

$$\frac{Q_2}{Q_1} = 0.05$$

i.e.

$$\left[\frac{1 - F_{11}}{1 - (1 - \varepsilon_1) \cdot F_{11}}\right] = 0.05$$

Solving,

$$1 - F_{11} = 0.05 - (0.05)^2 \cdot F_{11}$$

i.e. $$1 - 0.05 = F_{11} \cdot (1 - 0.0025)$$

i.e. $$F_{11} := \frac{0.95}{1 - 0.0025}$$

i.e. $$F_{11} = 0.952$$

But, we also have, for the view factor of cavity, w.r.t. itself:

$$F_{11} = 1 - \frac{A_2}{A_1}$$ *(where, A_2 is the area of the opening, A_1 is the area of surface of cavity)*

i.e. $$F_{11} = 1 - \frac{A_2}{A_s - A_2}$$ *(where, A_s = total area of spherical surface)*

Now, $A_s := \pi \cdot D^2$

i.e. $A_s = 0.071\ m^2$ *(total area of spherical surface)*

Therefore,

$$F_{11} = \left(1 - \frac{A_2}{A_s - A_2}\right) = 0.952$$

Solving,

$$\frac{A_s - A_2 - A_2}{A_s - A_2} = 0.952$$

i.e. $$A_s - 2 \cdot A_2 = 0.952 \cdot A_s - 0.952 \cdot A_2$$

i.e. $A_2 \cdot (2 - 0.952) = A_s \cdot (1 - 0.952)$

i.e. $1.048 \cdot A_2 = 3.393 \times 10^{-3}$

i.e. $$A_2 := \frac{3.393 \times 10^{-3}}{1.048}$$

i.e. $A_2 = 3.238 \times 10^{-3}\ m^2$ *(area of the opening on the surface of sphere)*

i.e. $A_2 = 32.38\ cm^2$ ***(area of the opening on the surface of sphere.)***

13.7.4 Radiation Heat Exchange in Three-zone Enclosures

Fig. 13.32 (a) shows an enclosure made of three opaque, diffuse, grey surfaces. Let the surfaces A_1, A_2, A_3 be maintained at uniform temperatures of T_1, T_2 and T_3, respectively. Also, let the emissivities be ε_1, ε_2 and ε_3, respectively. The radiation network for this system of three-surface enclosure is shown in Fig. 13.32 (b). While drawing the radiation network, the principle to be followed is quite simple: first, draw the surface resistance associated with each grey surface; then, connect the radiosity potentials between surfaces by the respective space resistances.

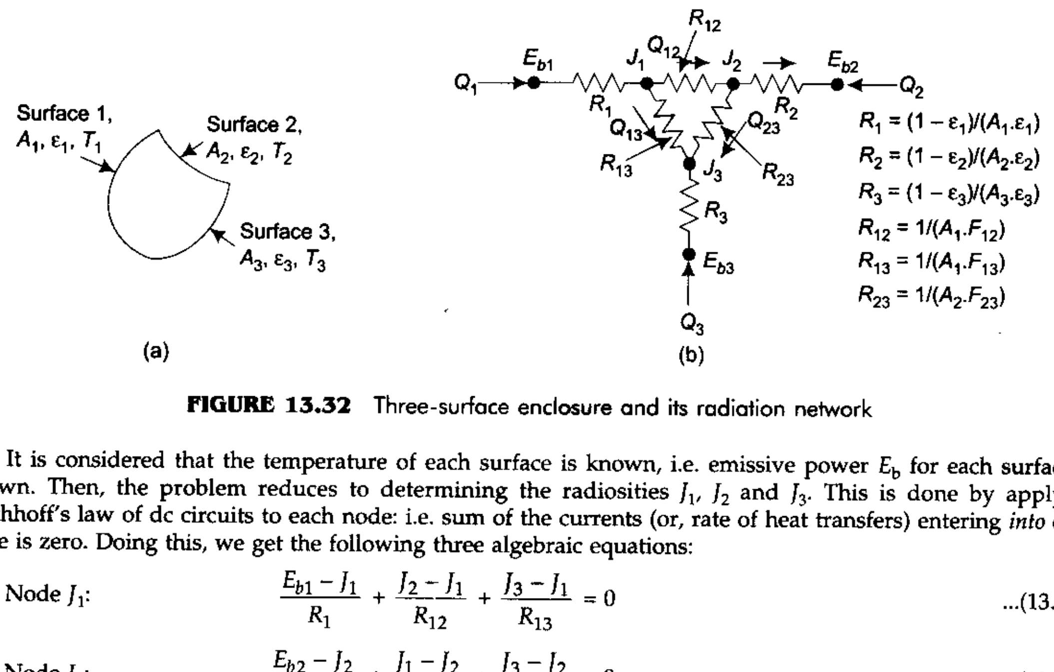


FIGURE 13.32 Three-surface enclosure and its radiation network

It is considered that the temperature of each surface is known, i.e. emissive power E_b for each surface is known. Then, the problem reduces to determining the radiosities J_1, J_2 and J_3. This is done by applying Kirchhoff's law of dc circuits to each node: i.e. sum of the currents (or, rate of heat transfers) entering *into* each node is zero. Doing this, we get the following three algebraic equations:

Node J_1: $$\frac{E_{b1} - J_1}{R_1} + \frac{J_2 - J_1}{R_{12}} + \frac{J_3 - J_1}{R_{13}} = 0 \quad \text{...(13.63a)}$$

Node J_2: $$\frac{E_{b2} - J_2}{R_2} + \frac{J_1 - J_2}{R_{12}} + \frac{J_3 - J_2}{R_{23}} = 0 \quad \text{...(13.63b)}$$

Node J_3: $$\frac{E_{b3} - J_3}{R_3} + \frac{J_1 - J_3}{R_{13}} + \frac{J_2 - J_3}{R_{23}} = 0 \quad \text{...(13.63c)}$$

Solving these three equations simultaneously, we get J_1, J_2 and J_3.

Remember to write each equation such that current flows *into* the node; then, the magnitudes of the radiosities would adjust themselves when all the three equations are solved simultaneously. Once the magnitudes of the radiosities are known, expressions for net heat flows between the surfaces are:

$$Q_{12} = \frac{J_1 - J_2}{R_{12}} = \frac{J_1 - J_2}{\dfrac{1}{A_1 \cdot F_{12}}} \quad \text{...(13.64a)}$$

$$Q_{13} = \frac{J_1 - J_3}{R_{13}} = \frac{J_1 - J_3}{\dfrac{1}{A_1 \cdot F_{13}}} \quad \text{...(13.64b)}$$

$$Q_{23} = \frac{J_2 - J_3}{R_{23}} = \frac{J_1 - J_2}{\dfrac{1}{A_2 \cdot F_{23}}} \quad \text{...(13.64c)}$$

and, net heat flow from each surface is:

$$Q_1 = \frac{E_{b1} - J_1}{R_1} = \frac{E_{b1} - J_1}{\left(\dfrac{1-\varepsilon_1}{A_1 \cdot \varepsilon_1}\right)} \quad \text{...(13.65a)}$$

$$Q_2 = \frac{E_{b2} - J_2}{R_2} = \frac{E_{b2} - J_2}{\left(\dfrac{1-\varepsilon_2}{A_2 \cdot \varepsilon_2}\right)} \quad \text{...(13.65b)}$$

$$Q_3 = \frac{E_{b3} - J_3}{R_3} = \frac{E_{b3} - J_3}{\left(\dfrac{1-\varepsilon_3}{A_3 \cdot \varepsilon_3}\right)} \quad \text{...(13.65c)}$$

Eq. set 13.64 is a set of general equations for three diffuse, opaque, grey surfaces. However, these equations will be modified depending upon any constraint that may be attached to any of the surfaces, i.e. say, if the surface is black or re-radiating: $J_i = E_{bi} = \sigma.T_i^4$. And, $Q_i = 0$ for a re-radiating surface. If Q_i at any surface is specified instead of the temperature (i.e. E_{bi}), then, $(E_{bi} - J_i)/R_i$ is replaced by Q_i.

We shall study a few such special cases of three-zone enclosures below:

Case (i): Two black surfaces connected to a third refractory surface:

This is a three-zone enclosure, with two of the surfaces being black and the third surface being a re-radiating, insulated surface. Typical example is a furnace whose bottom is the 'source' and the top is the 'sink' and the two surfaces are connected by a refractory wall which acts as a re-radiating surface. In effect, the source and sink exchange heat through the re-radiating wall; however, in steady state, the re-radiating wall radiates as much heat as it receives, which means that net heat exchange through the re-radiating wall (= Q) is zero, i.e. $E_b = J$ for the re-radiating wall. Therefore, once J (i.e. E_b) is calculated for the re-radiating surface, its steady state temperature can easily be calculated from: $E_b = \sigma.T^4$.

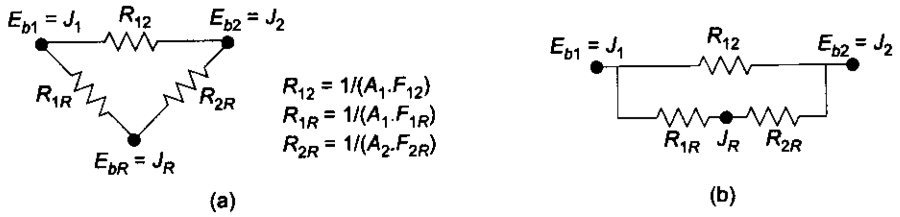


FIGURE 13.33 Two black surfaces connected by a third re-radiating surface and its radiation network

Fig. 13.33 (a) shows the radiation network for this case. The radiation network is drawn very easily by remembering the usual principles: for a black surface, the surface resistance is zero, i.e. $E_b = J$. For a re-radiating surface too, $E_b = J$, as already explained; further, for a re-radiating surface, $Q = 0$. Between two given surfaces, the radiosity potentials are connected by the respective space resistances, as shown. It may be observed that the system reduces to a series–parallel circuit of resistances as shown in Fig. 13.33 (b).

So, we write, for the total resistance of the circuit, R_{tot}:

$$\frac{1}{R_{tot}} = \frac{1}{R_{12}} + \frac{1}{(R_{1R} + R_{2R})}$$

and,

$$Q_{12} = \frac{E_{b1} - E_{b2}}{R_{tot}} = (E_{b1} - E_{b2}) \cdot \left[\frac{1}{R_{12}} + \frac{1}{(R_{1R} + R_{2R})}\right]$$

i.e. $$Q_{12} = \sigma \cdot (T_1^4 - T_2^4) \cdot \left[A_1 \cdot F_{12} + \frac{1}{\left(\frac{1}{A_1 \cdot F_{1R}} + \frac{1}{A_2 \cdot F_{2R}} \right)} \right] \quad \text{...(13.66)}$$

Here, Q_{12} is the net radiant heat transferred between surfaces 1 and 2. Similar expressions can be written for heat transfer between surfaces 2 and 3 (= Q_{23}) and the heat transfer between surfaces 1 and 3 (= Q_{13}).

Case (ii): Two grey surfaces surrounded by a third re-radiating surface:

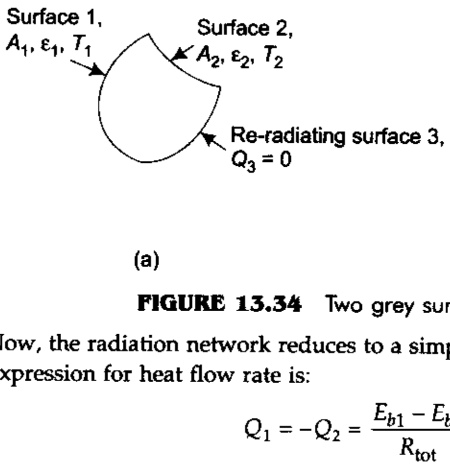
In this case, there are two grey surfaces, and the third surface is an insulated, re-radiating surface. As already explained, the re-radiating surface radiates as much energy as it receives; therefore, net radiant heat transfer for that surface is zero, i.e.

$$Q_3 = 0$$

i.e. $$\frac{E_{b3} - J_3}{\left(\frac{1 - \varepsilon_3}{A_3 \cdot \varepsilon_3} \right)} = 0$$

i.e. $$E_{b3} = J_3$$

i.e. once the radiosity of the re-radiating surface is known, its temperature can easily be calculated, since $E_{b3} = \sigma . T_3^4$. Further, note that T_3 is independent of the emissivity of surface 3.



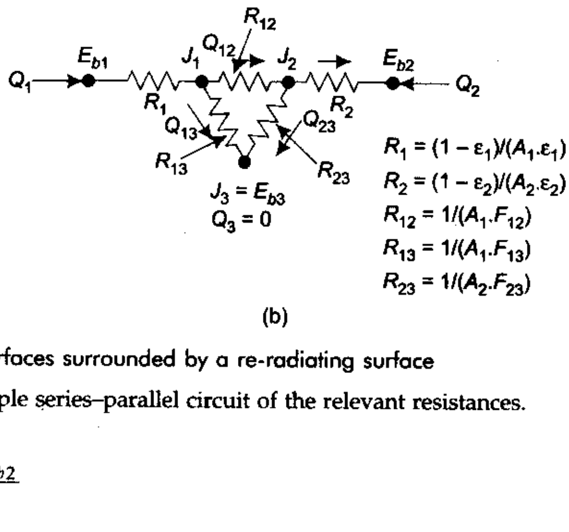


FIGURE 13.34 Two grey surfaces surrounded by a re-radiating surface

Now, the radiation network reduces to a simple series–parallel circuit of the relevant resistances.

Expression for heat flow rate is:

$$Q_1 = -Q_2 = \frac{E_{b1} - E_{b2}}{R_{tot}}$$

where, R_{tot} is the resistance, given by:

$$R_{tot} = R_1 + \left[\frac{1}{\frac{1}{R_{12}} + \frac{1}{(R_{13} + R_{23})}} \right] + R_2$$

i.e. $$R_{tot} = \left(\frac{1 - \varepsilon_1}{A_1 \cdot \varepsilon_1} \right) + \frac{1}{\left[A_1 \cdot F_{12} + \frac{1}{\frac{1}{\left(\frac{1}{A_1 \cdot F_{13}} + \frac{1}{A_2 \cdot F_{23}} \right)}} \right]} + \left(\frac{1 - \varepsilon_2}{A_2 \cdot \varepsilon_2} \right) \quad \text{...(13.67)}$$

13.7.5 Radiation Heat Exchange in Four-zone Enclosures

(a) **When all the four surfaces are black** Remembering the principles already explained, if the radiation network for an enclosure comprising of four black surfaces is drawn, it will look as shown in Fig. 13.35.

Expression for net radiant heat flow rate from surface 1 is:

$$Q_1 = \frac{J_1 - J_2}{R_{12}} + \frac{J_1 - J_3}{R_{13}} + \frac{J_1 - J_4}{R_{14}}$$

i.e. $$Q_1 = \frac{E_{b1} - E_{b2}}{R_{12}} + \frac{E_{b1} - E_{b3}}{R_{13}} + \frac{E_{b1} - E_{b4}}{R_{14}}$$

...*since* $E_{b1} = J_1$, etc.

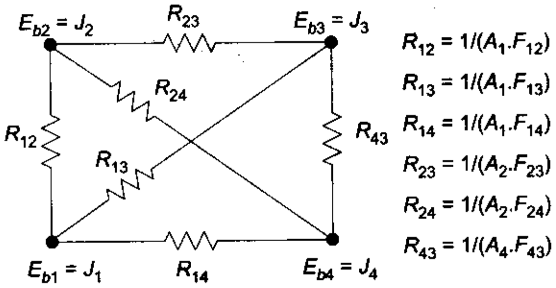


FIGURE 13.35 Radiation network for an enclosure of four black surfaces

i.e. $$Q_1 = A_1 \cdot F_{12} (E_{b1} - E_{b2}) + A_1 \cdot F_{13} \cdot (E_{b1} - E_{b3}) + A_1 \cdot F_{14} \cdot (E_{b1} - E_{b4}) \quad ...(13.68)$$

Similar expressions can be written for the net heat flow from other three surfaces.

(b) **When all the four surfaces are grey** Now, for each surface, a surface resistance also has to be included, and the radiation network for this system will be as shown in Fig. 13.36:

Expression for net radiant heat flow rate from surface 1 is:

$$Q_1 = \frac{E_{b1} - J_1}{R_1} = \frac{J_1 - J_2}{R_{12}} + \frac{J_1 - J_3}{R_{13}} + \frac{J_1 - J_4}{R_{14}}$$

i.e. $$Q_1 = \frac{E_{b1} - J_1}{\left(\dfrac{1 - \varepsilon_1}{A_1 \cdot \varepsilon_1}\right)} = A_1 \cdot F_{12} \cdot (J_1 - J_2) + A_1 \cdot F_{13} \cdot (J_1 - J_3) + A_1 \cdot F_{14} \cdot (J_1 - J_4) \quad ...(13.68)$$

Similar expressions can be written for the net heat flow from other three surfaces.

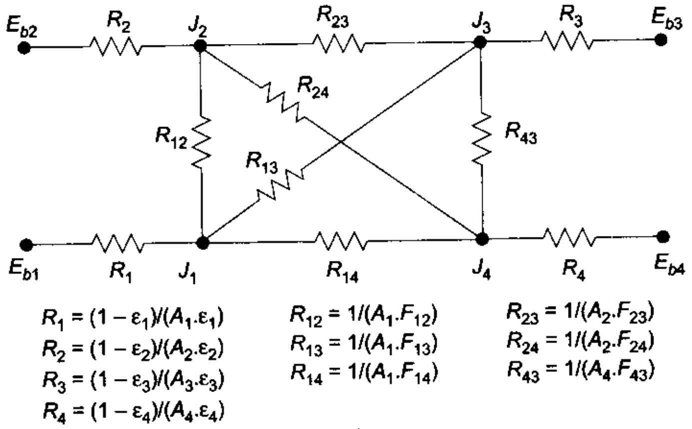


FIGURE 13.36 Radiation network for an enclosure of four grey surfaces

Example 13.22. A long duct of equilateral triangular section, of side w = 0.75 m, shown in Fig. Example 13.22, has its surface 1 at 700 K, surface 2 at 1000 K, and surface 3 is insulated. Further, surface 1 has an emissivity of 0.8 and surface 2 is black. Determine the rate at which energy must be supplied to surface 2 to maintain these operating conditions.

Solution. Since the duct is very long, the 'end effects' can be neglected. Therefore, this is a three zone enclosure, with surface 1 being grey, surface 2 being black, and surface 3 being insulated (or, re-rediating).

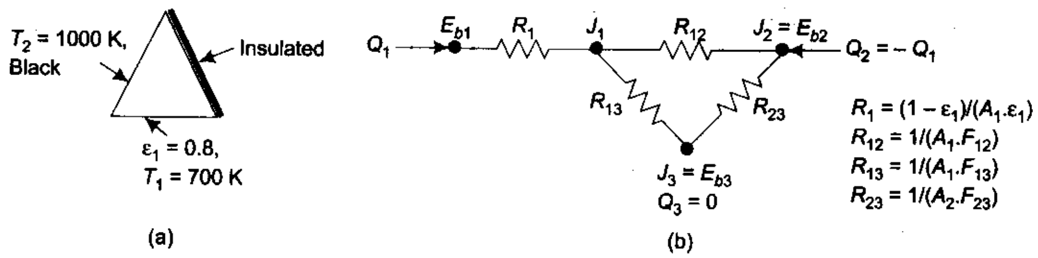


FIGURE Example 13.22 One black surface, one insulated surface, and one grey surface forming an enclosure, and its radiation network

Fig. Example 13.22 also shows the radiation network for this problem. This is drawn remembering the principles already stated, i.e. (a) for a black surface, the surface resistance is zero, and $E_b = J$, (b) for an insulated (or re-radiating) surface, $Q = 0$ and $J = E_b$, (c) for a grey surface, add a surface resistance, $(1 - \varepsilon)/(A.\varepsilon)$, and (d) connect the radiosity potentials by the respective space resistances $(1/A_i \cdot F_{ij})$.

Data:

Let the length of the duct be 1 m

i.e. $L := 1$ m (length of duct) $\quad W := 0.75$ m (side of equilateral triangle) $\quad T_1 := 700$ K $\quad \varepsilon_1 := 0.8$

$T_2 := 1000$ K $\quad \sigma := 5.67 \times 10^{-8}$ W/(m²K) (Stefan–Boltzmann constant)

Now, we have, for view factors:

$$F_{11} + F_{12} + F_{13} = 1 \qquad \textit{(by summation rule)}$$

But, $F_{11} = 0$ *(since surface 1 is flat, and cannot 'see' itself.)*

Then, $F_{12} + F_{13} = 1$

Further, by symmetry, $F_{12} = F_{13}$ for equilateral triangle.

Therefore, $F_{12} = 0.5$

and, $F_{13} = 0.5$

Similarly, $F_{23} = 0.5$

Since surface 3 is re-radiating surface, net heat transfer for that surface $Q_3 = 0$.

Therefore, $Q_1 = -Q_2$

And, radiation network is a simple series–parallel network as shown in Fig. Example 13.22 (b) above. Then, Q_1 is determined directly as:

$$Q_1 = \frac{E_{b1} - E_{b2}}{R_1 + \left(\dfrac{1}{R_{12}} + \dfrac{1}{R_{13} + R_{23}}\right)^{-1}} \text{ W} \qquad \text{...(a)}$$

i.e.

$$Q_1 = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{A_1 \cdot \varepsilon_1} + \left(A_1 \cdot F_{12} + \dfrac{1}{\dfrac{1}{A_1 \cdot F_{13}} + \dfrac{1}{A_2 \cdot F_{23}}}\right)^{-1}}$$

Areas:

$A_1 := W \cdot L$ i.e. $A_1 = 0.75$ m² *(area of surface 1)*

and, for equilateral triangle:

$A_2 := A_1$

and, $A_3 := A_1$

Resistances:

Surface resistance: $R_1 := \dfrac{1-\varepsilon_1}{A_1 \cdot \varepsilon_1}$

i.e. $R_1 = 0.333$ m^{-2}

Space resistances:

$$R_{12} := \frac{1}{A_1 \cdot F_{12}}$$

i.e. $R_{12} = 2.667\ \text{m}^{-2}$

and, $R_{13} := \dfrac{1}{A_1 \cdot F_{13}}$

i.e. $R_{13} = 2.667\ \text{m}^{-2}$

and, $R_{23} := \dfrac{1}{A_2 \cdot F_{23}}$

i.e. $R_{23} = 2.667\ \text{m}^{-2}$

Therefore, from Eq. a:

$$Q_1 := \frac{\sigma \cdot (T_1^4 - T_2^4)}{R_1 + \left(\dfrac{1}{R_{12}} + \dfrac{1}{R_{13} + R_{23}}\right)^{-1}}\ \text{W}. \qquad \text{(a)}$$

i.e. $Q_1 = -2.041 \times 10^4\ \text{W}$

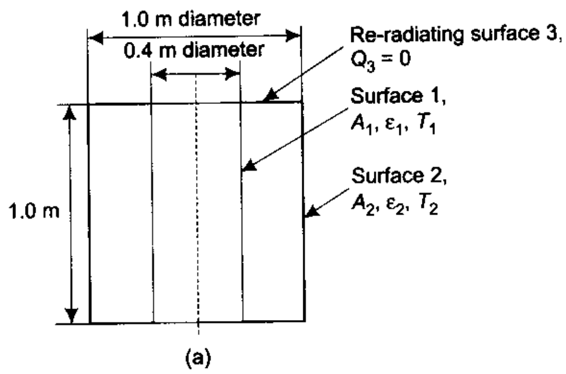
and, $Q_2 = -Q_1 = 2.041 \times 10^4\ \text{W}$ ***(energy to be supplied to heated surface 2 per metre length.)***

Example 13.23. Two co-axial cylinders of 0.4 m and 1 m diameter are 1 m long. The annular top and bottom surfaces are well insulated and act as re-radiating surfaces. The inner surface is at 1000 K and has an emissivity of 0.6. The outer surface is maintained at 400 K and its emissivity is 0.4.

(i) Determine the heat exchange between the surfaces

(ii) If the annular base surfaces are open to the surroundings at 300 K, determine the radiant heat exchange.

If the outer cylinder is surface 2, take F_{21}= 0.25 and F_{22}= 0.27. (M.U. Dec. 1998)



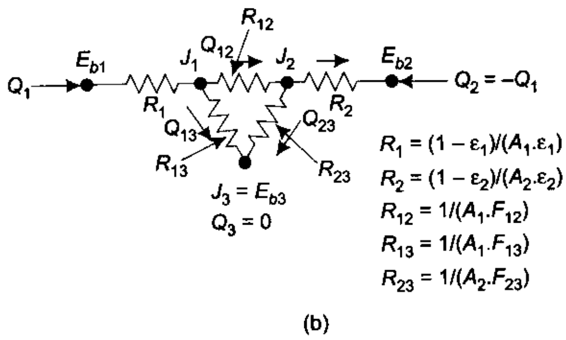


FIGURE Example 13.23 Two gray surfaces surrounded by a re-radiating surface

Solution. See Fig. Example 13.23. Let the inner surface be denoted by 1, outer surface by 2, and the two annular surfaces by 3. Then, surfaces 1, 2 and 3 form an enclosure. And, the rediation network will look as shown in the Fig. Example 13.23.

Data:

$D_1 := 0.4$ m $\quad D_2 := 1$ m $\quad L := 1$ m $\quad T_1 := 1000$ K $\quad T_2 := 400$ K $\quad \varepsilon_1 := 0.6 \quad \varepsilon_2 := 0.4$

$F_{21} := 0.25 \quad F_{22} := 0.27$

$\sigma := 5.67 \times 10^{-8}\ \text{W/(m}^2\text{K)}$ (Stefan–Boltzmann constant)

Areas:

$A_1 := \pi \cdot D_1 \cdot L$ i.e. $A_1 = 1.257\ \text{m}^2$ *(surface area of inner cylinder 1)*

$A_2 := \pi \cdot D_2 \cdot L$ i.e. $A_2 = 3.142\ \text{m}^2$ *(surface area of outer cylinder 2)*

To find F_{12}:

$F_{11} = 0$ *(since surface 1 is convex, and does not 'see' itself.)*

Then, $F_{12} := \dfrac{A_2}{A_1} \cdot F_{21}$ *(by reciprocity)*

i.e. $F_{12} = 0.625$ *(view factor from surface 1 to surface 2)*

Also, $F_{11} + F_{12} + F_{13} = 1$ *(by summation rule)* i.e. $F_{13} := 1 - (F_{11} + F_{12})$

i.e. $F_{13} = 0.375$ *(view factor from surface 1 to surface 3)*

Also, $F_{21} + F_{22} + F_{23} = 1$ *(by summaion rule)*

i.e. $F_{23} := 1 - (F_{21} + F_{22})$

i.e. $F_{23} = 0.48$ *(view factor from surface 2 to surface 3)*

Emissive powers:

$E_{b1} := \sigma \cdot T_1^4$ i.e. $E_{b1} = 5.67 \times 10^4 \text{ W/m}^2$ *(Emissive power of surface 1)*

$E_{b2} := \sigma \cdot T_2^4$ i.e. $E_{b2} = 1.452 \times 10^3 \text{ W/m}^2$.*(Emissive power of surface 2)*

Resistances:

$R_1 := \dfrac{1-\varepsilon_1}{A_1 \cdot \varepsilon_1}$ i.e. $R_1 = 0.531 \text{ m}^{-2}$ *(surface resistance of inner cylinder 1)*

$R_2 := \dfrac{1-\varepsilon_2}{A_2 \cdot \varepsilon_2}$ i.e. $R_2 = 0.477 \text{ m}^{-2}$ *(surface resistance of outer cylinder 2)*

$R_{12} := \dfrac{1}{A_1 \cdot F_{12}}$ i.e. $R_{12} = 1.273 \text{ m}^{-2}$ *(space resistance between surfaces 1 and 2)*

$R_{13} := \dfrac{1}{A_1 \cdot F_{13}}$ i.e. $R_{13} = 2.122 \text{ m}^{-2}$ *(space resistance between surfaces 1 and 3)*

$R_{23} := \dfrac{1}{A_2 \cdot F_{23}}$ i.e. $R_{23} = 0.663 \text{ m}^{-2}$ *(space resistance between surfaces 2 and 3)*

Case (i): When both the annular surfaces act as re-radiating surfaces:

We have: $Q = \dfrac{E_{b1} - E_{b2}}{R_1 + R_{eff} + R_2}$ W *(heat exchange between the surfaces)*

The radiation network is as shown above. For the series–parallel network of resistances, we observe that R_{12} and $(R_{13} + R_{23})$ are in parallel. Therefore, effective resistance R_{eff} is given by:

$$\frac{1}{R_{eff}} = \frac{1}{R_{12}} + \frac{1}{R_{13} + R_{23}}$$

i.e. $$R_{eff} := \left(\frac{1}{R_{12}} + \frac{1}{R_{13} + R_{23}}\right)^{-1}$$

i.e. $R_{eff} = 0.874 \text{ m}^{-2}$ *(effective resistance)*

Therefore,

$$Q := \frac{E_{b1} - E_{b2}}{R_1 + R_{eff} + R_2} \text{ W}$$ *(heat exchange between the surfaces)*

i.e. $Q = 2.936 \times 10^4$ **W** ***(heat exchange between the surfaces.)***

In addition, for case (i), if we wish to determine the temperature of re-radiating surface:

Apply the condition that for re-radiating surface, in steady state,
heat received by the surface = heat lost by the surface

i.e. $$\frac{J_1 - J_3}{R_{13}} = \frac{J_3 - J_2}{R_{23}}$$

So, we have to determine J_1 and J_2.

Now, $Q = Q_1 = -Q_2$

We have: $Q_1 = \dfrac{E_{b1} - J_1}{R_1}$ and, $Q_1 := Q$

i.e. $J_1 := E_{b1} - Q_1 \cdot R_1$

i.e. $J_1 = 4.112 \times 10^4 \text{ W/m}^2$

and,

$Q_2 = \dfrac{E_{b2} - J_2}{R_2}$ and, $Q_2 := -Q_1$

i.e. $J_2 := E_{b2} - Q_2 \cdot R_2$

i.e. $J_2 = 1.547 \times 10^4 \text{ W/m}^2$

Now, $\dfrac{J_1 - J_3}{R_{13}} = \dfrac{J_3 - J_2}{R_{23}}$ *(for re-radiating surface)*

Therefore, $J_3 \cdot (R_{13} + R_{23}) = J_1 \cdot R_{23} + J_2 \cdot R_{13}$

i.e. $J_3 := \dfrac{J_1 \cdot R_{23} + J_2 \cdot R_{13}}{R_{13} + R_{23}}$

i.e. $J_3 = 2.158 \times 10^4 \text{ W/m}^2$

But, $J_3 = E_{b3} = \sigma \cdot T_3^4$

Therefore, $T_3 := \left(\dfrac{J_3}{\sigma}\right)^{\frac{1}{4}}$

i.e. $T_3 = 785.429 \text{ K}$ ***(equilibrium temperature of re-radiating surface.)***

Case (ii): When both the annular surfaces are open to surroundings at 300 K:

Now, $T_{surr} := 300 \text{ K}$ *(temperature of surroundings)*

Further, $E_{b3} = \sigma \cdot T_{surr}^4$

i.e. $E_{b3} = 459.27 \text{ W/m}^2$ *(emissive power of surface 3 (i.e. surroundings))*

and, $J_3 := E_{b3}$

To find J_1 and J_2 Apply Kirchhoff's law to nodes J_1 and J_2:

At J_1: $\dfrac{E_{b1} - J_1}{R_1} + \dfrac{J_2 - J_1}{R_{12}} + \dfrac{J_3 - J_1}{R_{13}} = 0$ (a)

At J_2: $\dfrac{E_{b2} - J_2}{R_2} + \dfrac{J_1 - J_2}{R_{12}} + \dfrac{J_3 - J_2}{R_{23}} = 0$ (b)

To get values of J_1 and J_2 solve Eqs. a and b simultaneously:

We shall use solve block of Mathcad to solve Eqs. a and b:

First, choose trial (or, guess) values for J_1 and J_2. Then, immediately after 'Given', write the constraints, i.e. Eqs. a and b. Now, type 'Find (J_1, J_2) = ', and the result appears immediately:

$J_1 := 100 \quad J_2 := 100$ *(trial values)*

Given

$$\frac{E_{b1} - J_1}{R_1} + \frac{J_2 - J_1}{R_{12}} + \frac{J_3 - J_1}{R_{13}} = 0$$

$$\frac{E_{b2} - J_2}{R_2} + \frac{J_1 - J_2}{R_{12}} + \frac{J_3 - J_2}{R_{23}} = 0$$

$$\text{Find}(J_1, J_2) = \begin{bmatrix} 3.591 \times 10^4 \\ 7.278 \times 10^3 \end{bmatrix}$$

i.e. $J_1 := 3.591 \times 10^4 \text{ W/m}^2$

and, $J_2 := 7.278 \times 10^3 \text{ W/m}^2$

Therefore, heat lost by surface 1:

$$Q_1 := \frac{E_{b1} - J_1}{R_1}$$

i.e. $Q_1 = 3.919 \times 10^4$ **W.**

And, heat lost by surface 2:

$$Q_2 := \frac{E_{b2} - J_2}{R_2}$$

i.e. $Q_2 = -1.22 \times 10^4$ W.

Note that negative sign indicates that flow is into the surface.

Heat gained by surroundings:

$$Q_s = \frac{J_1 - J_3}{R_{13}} + \frac{J_2 - J_3}{R_{23}}$$

i.e. $$Q_s = 2.699 \times 10^4 \text{ W}$$

Verify:

Heat gained by the surroundings must be equal to hea lost by the surfaces.

i.e. $$Q_s = Q_1 + Q_2$$

$$Q_1 + Q_2 = 2.699 \times 10^4 = Qs$$ *(verified.)*

Example 13.24. Two parallel plates, 0.5 m × 1 m each, are spaced 0.5 m apart. The plates are at temperatures of 1000°C and 500°C and their emissivities are 0.2 and 0.5, respectively. The plates are located in a large room, the walls of which are at 27°C. The surfaces of the plates facing each other only exchange heat by radiation. Determine the rates of heat lost by each plate and heat gain of the walls by radiation. Use radiation network for solution.

Assume shape factor between parallel plates: $F_{12} = F_{21} = 0.285$. (M.U. 1996)

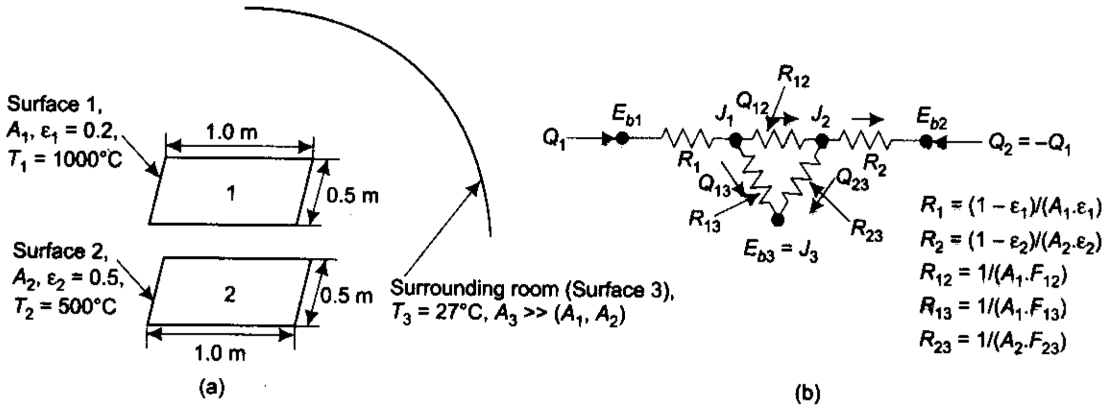


FIGURE Example 13.24 Two grey surfaces surrounded by a large room

Solution.

This is a three-zone enlosure, and the radiation network for this system is shown in Fig. Example 13.24 (b) above. Since the area A_3 of the room is very large, we can take the surface resistance of A_3 as equal to zero.

i.e. $$\frac{1-\varepsilon_3}{A_3 \cdot \varepsilon_3} = 0$$

This means that $E_{b3} = J_3$, i.e. a large room is equivalent to a black surface.

Data:

$A_1 := 0.5 \text{ m}^2$ $A_2 := 0.5 \text{ m}^2$ $T_1 := 1000 + 273 \text{ K}$ $T_2 := 500 + 273 \text{ K}$ $T_3 := 27 + 273 \text{ K}$ $\varepsilon_1 := 0.2$

$\varepsilon_2 := 0.5$ $F_{12} := 0.285$ $F_{21} := 0.285$ $\sigma := 5.67 \times 10^{-8} \text{ W/(m}^2\text{K)}$ (Stefan–Boltzmann constant)

Now, $F_{11} + F_{12} + F_{13} = 1$ *(by summation rule)*

But, $F_{11} = 0$ *(since surface 1 is flat and can not 'see' itself.)*

Therefore, $F_{12} + F_{13} = 1$

and, $F_{13} := 1 - F_{12}$

i.e. $F_{13} = 0.715$ *(view factor of surface 1 w.r.t. surface 3)*

Similarly, $F_{23} := 0.715$ *(view factor of surface 2 w.r.t. surface 3)*

Resistances:

$$R_1 := \frac{1-\varepsilon_1}{A_1 \cdot \varepsilon_1} \quad \text{i.e.} \quad R_1 = 8 \text{ m}^{-2}$$ *(surface resistance of surface 1)*

$$R_2 := \frac{1-\varepsilon_2}{A_2 \cdot \varepsilon_2} \quad \text{i.e.} \quad R_2 = 2 \text{ m}^{-2}$$ *(surface resistance of surface 2)*

$$R_{12} := \frac{1}{A_1 \cdot F_{12}} \quad \text{i.e.} \quad R_{12} = 7.018 \text{ m}^{-2}$$ *(space resistance between surfaces 1 and 2)*

$R_{13} := \dfrac{1}{A_1 \cdot F_{13}}$ i.e. $R_{13} = 2.797\ \text{m}^{-2}$ *(space resistance between surfaces 1 and 3)*

$R_{23} := \dfrac{1}{A_2 \cdot F_{23}}$ i.e. $R_{23} = 2.797\ \text{m}^{-2}$ *(space resistance between surfaces 2 and 3)*

Heat lost by each surface:

$$Q_1 = \frac{E_{b1} - J_1}{R_1} = \text{heat lost by surface 1}$$

and,

$$Q_2 = \frac{E_{b2} - J_2}{R_2} = \text{heat lost by surface 2}$$

And, heat gain by surface 3:

$$Q_3 = Q_{13} + Q_{23}$$

i.e.

$$Q_3 = \frac{J_1 - J_3}{R_{13}} + \frac{J_2 - J_3}{R_{23}}$$

Therefore, the problem reduces to calculating the radiosities, J_1, J_2 and J_3.

To calculate the radiosities J_1 and J_2, apply Kirchhoff's law of electric circuits of nodes J_1 and J_2:

Node J_1:

$$\frac{E_{b1} - J_1}{R_1} + \frac{J_2 - J_1}{R_{12}} + \frac{E_{b3} - J_1}{R_{13}} = 0 \qquad \text{(a)}$$

Node J_2:

$$\frac{J_1 - J_2}{R_{12}} + \frac{E_{b3} - J_2}{R_{23}} + \frac{E_{b2} - J_2}{R_2} = 0 \qquad \text{(b)}$$

Emissive powers:

$E_{b1} := \sigma \cdot T_1^4$ i.e. $E_{b1} = 1.489 \times 10^5\ \text{W/m}^2$ *(for surface 1)*

$E_{b2} := \sigma \cdot T_2^4$ i.e. $E_{b2} = 2.024 \times 10^4\ \text{W/m}^2$ *(for surface 2)*

$E_{b3} := \sigma \cdot T_3^4$ i.e. $E_{b3} = 459.27\ \text{W/m}^2$ *(for surface 3)*

Note that: $J_3 := E_{b3}$ *(for the large room.)*

To get J_1 and J_2, solve Eqs. a and b simultaneously. To do this, we shall use solve block of Mathcad.

First, choose trial (or, guess) values for J_1 and J_2. Then, immediately after 'Given', write the constraints i.e. Eqs. a and b. Now, type 'Find (J_1, J_2) = ', and the result appears immediately:

$J_1 := 100$ $J_2 := 100$ *...trial values*

Given

$$\frac{E_{b1} - J_1}{R_1} + \frac{J_2 - J_1}{R_{12}} + \frac{E_{b3} - J_1}{R_{13}} = 0$$

$$\frac{J_1 - J_2}{R_{12}} + \frac{E_{b3} - J_2}{R_{23}} + \frac{E_{b2} - J_2}{R_2} = 0$$

$$\text{Find}(J_1, J_2) = \begin{bmatrix} 3.3476 \times 10^4 \\ 1.5057 \times 10^4 \end{bmatrix}$$

i.e. $J_1 := 3.3476 \times 10^4\ \text{W/m}^2$

and, $J_2 := 1.5057 \times 10^4\ \text{W/m}^2$

Therefore,

Heat lost by each surface:

$$Q_1 := \frac{E_{b1} - J_1}{R_1}$$

i.e. $Q_1 = 1.443 \times 10^4\ \text{W}$ **(= *heat lost by surface* 1.)**

And,

$$Q_2 := \frac{E_{b2} - J_2}{R_2}$$

i.e. $Q_2 = 2.594 \times 10^3\ \text{W}$ **(= *heat lost by surface* 2.)**

Now, heat lost by both surfaces 1 and 2 is gained by the surroundings; so, heat gained by surroundings = $Q_3 = Q_1 + Q_2$

i.e. $Q_3 := Q_1 + Q_2$

i.e. $Q_3 = 1.702 \times 10^4$ W (= *heat gained by surface* 3.)

Verify:

We have: $Q_3 := \left(\frac{J_1 - J_3}{R_{13}} + \frac{J_2 - J_3}{R_{23}}\right)$

i.e. $Q_3 = 1.702 \times 10^4$ W (= *heat gained by surface 3...verified.*)

13.8 Radiation Shielding

In practice, quite often, one or more 'radiation shields' are used to reduce radiant heat transfer between two given surfaces. Radiation shield is, simply a thin, high reflectivity surface placed in between the surfaces which exchange heat between themselves. Radiation shields may be made of aluminium foils, copper foils, or aluminised mylar sheets, etc. Radiation shields are extensively used in building industry to reduce radiant heat loss from or to the walls; in cryogenic industry as 'super-insulation' (i.e. alternate layers of an insulator and reflector, e.g. glass fibre mat + aluminium foils or, aluminised mylar sheets, about 25 numbers per inch) to reduce the heat leakage into cryogenic vessels or cryostats, in space industry, again as 'super-insulation' to reduce heat in-leaks, etc. Radiation shield does not participate in heat transfer, i.e. it does not add or remove heat from the system as such, but reduces the heat transfer by interposing additional 'resistance' in the path of heat transfer.

We shall study how the heat transfer is reduced by the use of radiation shields, with reference to two infinite, parallel plates, which exchange heat between themselves.

Fig. 13.37 (a) shows two large parallel plates, 1 and 2 exchanging heat between themselves; let their areas, temperatures (in Kelvin) and emissivities be (A_1, T_1, ε_1) and (A_2, T_2, ε_2). Let a radiation shield 3, be placed between these plates. Plate 3 is thin and made of a material of high reflectivity. Let the emissivities of two sides of the radiation shield be ε_{3-1} and ε_{3-2} as shown. Radiation network for this system is shown in Fig. 13.37 (b). This is drawn, as usual, remembering that each grey surface has a 'surface resistance' associated with it, and the two radiosity potentials are connected by a 'space resistance'.

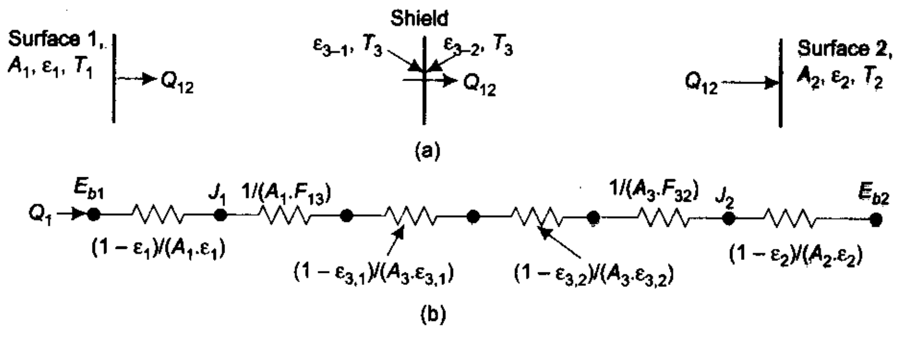


FIGURE 13.37 Radiation shield between two parallel plates, and associated radiation network

When there is no shield, the radiation heat transfer between plates 1 and 2 is already shown to be:

$$Q_{12} = \frac{A \cdot \sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$ (*for infinitely large parallel plates...*(13.59))

With one shield placed between plates 1 and 2, the radiation network will be as shown in Fig. 13.17 (b) above. Note that now all the relevant resistances are in series. Net heat transfer between plates 1 and 2 is given as:

$Q_{12\_one\ shield} = (E_{b1} - E_{b2})/R_{tot}$ where, R_{tot} is the total resistance.

i.e.

$$Q_{12\_one\_shield} = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{A_1 \cdot \varepsilon_1} + \frac{1}{A_1 \cdot F_{13}} + \frac{1-\varepsilon_{3\_1}}{A_3 \cdot \varepsilon_{3\_1}} + \frac{1-\varepsilon_{3\_2}}{A_3 \cdot \varepsilon_{3\_2}} + \frac{1}{A_3 \cdot F_{32}} + \frac{1-\varepsilon_2}{A_2 \cdot \varepsilon_2}}$$

(for two grey surfaces with one radiation shield placed in between....(13.70))

Now, for two large parallel plates, we note:

$F_{13} = F_{32} = 1$ and, $A_1 = A_2 = A_3 = A$

Then, Eq. 13.70 simplifies to:

$$Q_{12\_one\_shield} = \frac{A \cdot \sigma \cdot (T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3\_1}} + \frac{1}{\varepsilon_{3\_2}} - 1\right)} \quad ...(13.71)$$

Note that as compared to Eq. 13.59 for the case of no-shield, we have, with one shield, an additional term appearing in the denominator of Eq. 13.71. Therefore, if there are N radiation shields, we have, for net radiation heat transfer:

$$Q_{12_{N\_shields}} = \frac{A \cdot \sigma \cdot (T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3\_1}} + \frac{1}{\varepsilon_{3\_2}} - 1\right) + \dots + \left(\frac{1}{\varepsilon_{N\_1}} + \frac{1}{\varepsilon_{N\_2}} - 1\right)} \quad ...(13.72)$$

If emissivities of all surfaces are equal, Eq. 13.72 becomes:

$$Q_{12_{N\_shields}} = \frac{A \cdot \sigma \cdot (T_1^4 - T_2^4)}{(N+1) \cdot \left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon} - 1\right)} = \frac{1}{(N+1)} \cdot (Q_{12_{no\_shield}}) \quad ...(13.73)$$

Note this important result, which implies that, when all emissivities are equal, presence of one radiation shield reduces the radiation heat transfer between the two surfaces to one-half, two radiation shields reduce the heat transfer to one-third, 9 radiation shields reduce the heat transfer to one-tenth, etc.

For a more practical case of the two surfaces having emissivities of ε_1 and ε_2, and all shields having the same emissivity of ε_s, Eq. 13.72 becomes:

$$Q_{12_{N\_shields}} = \frac{A \cdot \sigma \cdot (T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + N \cdot \left(\frac{2}{\varepsilon_s} - 1\right)} \quad ...(13.74)$$

To determine the equilibrium temperature of the radiation shield:

Once Q_{12} is determined from Eq. 13.71, the temperature of the shield is easily found out by applying the condition that in steady state:

$$Q_{12} = Q_{13} = Q_{32} \quad ...(13.75)$$

We can use either of the conditions: $Q_{12} = Q_{13}$ or $Q_{12} = Q_{32}$.

Q_{13} or Q_{32} is determined by applying Eq. 13.59; i.e. we get:

$$Q_{12} = Q_{13} = \frac{A \cdot \sigma \cdot (T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} \quad ...(13.76a)$$

or,

$$Q_{12} = Q_{32} = \frac{A \cdot \sigma \cdot (T_3^4 - T_2^4)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1} \quad ...(13.76b)$$

In both the above equations, T_3 is the only unknown, which can easily be determined.

For a cylindrical radiation shield placed in between two, long concentric cylinders:

Consider the case of radiation heat transfer between two long, concentric cylinders. The radiation heat transfer between two long, concentric cylinders is already shown to be:

$$Q_{12} = \frac{A \cdot \sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right)}$$

...for infinitely long concentric cylinders...(13.60)

where,

$$\frac{A_1}{A_2} = \frac{r_1}{r_2}$$

Now, let a cylindrical radiation shield, 3, be placed in between the inner cylinder (1) and the outer cylinder (2), as shown in Fig. 13.38.

The radiation network for this system is shown in Fig. 13.38 (b) and it is exactly the same as shown in Fig. 13.37 (b). And, the radiation heat transfer between cylinders 1 and 2, when the shield is present, is given by:

$$Q_{12\_one\_shield} = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{A_1 \cdot \varepsilon_1} + \frac{1}{A_1 \cdot F_{13}} + \frac{1-\varepsilon_{3\_1}}{A_3 \cdot \varepsilon_{3\_1}} + \frac{1-\varepsilon_{3\_2}}{A_3 \cdot \varepsilon_{3\_2}} + \frac{1}{A_3 \cdot F_{32}} + \frac{1-\varepsilon_2}{A_2 \cdot \varepsilon_2}}$$

(*for two grey surfaces with one radiation shield placed in between*...(13.70))

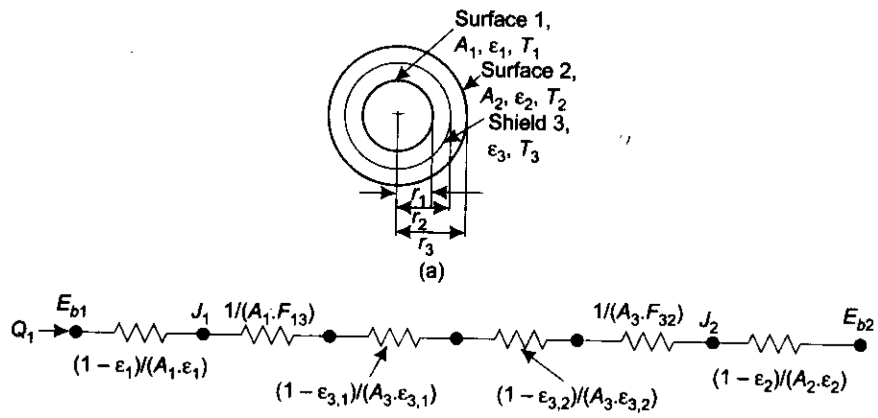


FIGURE 13.38 Radiation shield between two concentric cylinders, and associated radiation network

Now, for the cylindrical system, we have:

$$F_{13} = F_{32} = 1$$
$$A_1 = 2 \cdot \pi \cdot r_1 \cdot L$$
$$A_2 = 2 \cdot \pi \cdot r_2 \cdot L$$

and,

$$A_3 = 2 \cdot \pi \cdot r_3 \cdot L$$

Then, Eq. 13.70 reduces to:

$$Q_{12one\_shield} = \frac{A_1 \cdot \sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right) + \left(\frac{A_1}{A_3}\right) \cdot \left(\frac{1}{\varepsilon_{3\_1}} + \frac{1}{\varepsilon_{3\_2}} - 1\right)}$$

(*for concentric cylinders with one radiation shield*...(13.77))

In Eq. 13.77, we have: $(A_1/A_2) = (r_1/r_2)$, and $(A_1/A_3) = (r_1/r_3)$.

Note that as compared to the relation for two concentric cylinders with no shield (i.e. Eq. 13.60), an additional term appears in the denominator of Eq. 13.77 (i.e. the third term) when one radiation shield is introduced; if there is a second radiation shield, say (4), then one more similar term will have to be added in the denominator to take care of the resistance of that shield.

In this case, too, the equilibrium temperature of the shield is determined by applying the principle that, in steady state,

$$Q_{12} = Q_{13} = Q_{32}.$$

For a spherical radiation shield placed in between two concentric spheres:

This case is also represented by Fig. 13.38 (a), where inner sphere 1 is enclosed by an outer sphere 2, and a radiation shield 3, is placed in between. The radiation network for this system is shown in Fig. 13.38 (b).

When there is no radiation shield, radiation heat transfer between surfaces 1 and 2 is given by Eq. 13.61, i.e.

$$Q_{12} = \frac{A_1 \cdot \sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right)}$$ *(for concentric spheres...(13.61))*

where,

$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2$$

Again, when the radiation shield is present, the general relation for radiation heat transfer between surfaces 1 and 2 is Eq. 13.70. Remembering that for concentric spheres,

$$F_{13} = F_{32} = 1$$
$$A_1 = 4 \cdot \pi \cdot r_1^2$$
$$A_2 = 4 \cdot \pi \cdot r_2^2$$

and, $$A_3 = 4 \cdot \pi \cdot r_3^2$$

relation for radiant heat transfer between surfaces 1 and 2, is exactly as Eq. 13.77, i.e.

$$Q_{12one\_shield} = \frac{A_1 \cdot \sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right) + \left(\frac{A_1}{A_3}\right) \cdot \left(\frac{1}{\varepsilon_{3\_1}} + \frac{1}{\varepsilon_{3\_2}} - 1\right)}$$

(for concentric spheres with one radiation shield...(13.78))

In Eq. 13.78, we have:

$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2 \text{ and, } \frac{A_1}{A_3} = \left(\frac{r_1}{r_3}\right)^2$$

In this case also, equilibrium temperature of the shield is determined by applying the principle that, in steady state,

$$Q_{12} = Q_{13} = Q_{32}.$$

Example 13.25. Two large parallel planes facing each other and having emissivities 0.3 and 0.5 are maintained at 827°C and 527°C, respectively. Determine the rate at which heat is exchanged between the two surfaces by radiation. If a radiation shield of emissivity 0.05 on both sides is placed parallel between the two surfaces, determine the percentage reduction in the radiant heat exchange rate. (M.U., Jan. 2002)

Solution. This is the case of one radiation shield placed in between two parallel plates. See Fig. Example 13.25.

Data:

$T_1 := 827 + 273$ K $\quad T_2 := 527 + 273$ K $\quad \varepsilon_1 := 0.3 \quad \varepsilon_2 := 0.5 \quad \varepsilon_{31} := 0.05 \quad \varepsilon_{32} := 0.05$

$\sigma := 5.67 \times 10^{-8}$ W/(m²K) *(Stefan–Boltzmann constant)* $\quad A := 1$ m² *(surface area of plates...assumed)*

(a) **Heat exchange between surfaces 1 and 2, when there is no shield:**

$$Q_{12} := \frac{A \cdot \sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \text{ W}$$ *(for infinitely large parallel plates...(13.59))*

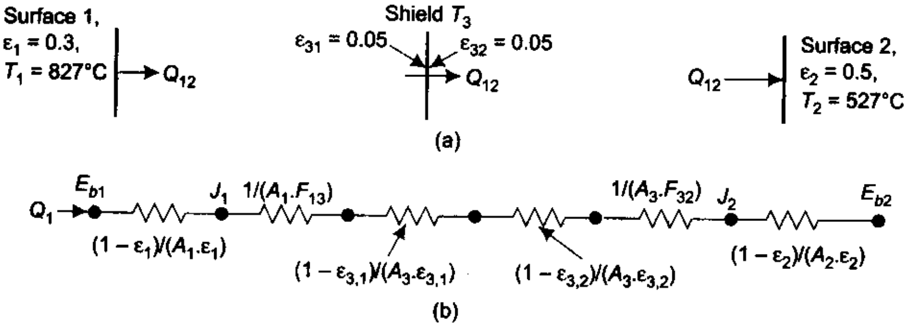


FIGURE Example 13.25 Radiation shield between two parallel plates, and associated radiation network

i.e. $Q_{12} = 1.38 \times 10^4$ **W/m²** ***(radiant heat transfer, without shield.)***

Now, we have for radiant heat transfer between surfaces 1 and 2; when there is one shield in between 1 & 2:

$$Q_{12\_one\_shield} := \frac{A \cdot \sigma \cdot (T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{31}} + \frac{1}{\varepsilon_{32}} - 1\right)} \quad ...(13.71)$$

i.e. $Q_{12\_one\_shield} = 1.38 \times 10^3$ **W/m²** ***(radiant heat transfer, with one shield.)***

Therefore, percentage reduction in heat transfer due to radiation shield:

$$\text{Reduction} = \frac{Q_{12} - Q_{12\_one\_shield}}{Q_{12}} \cdot 100 = \frac{(1.38 \times 10^4 - 1.38 \times 10^3)}{1.38 \times 10^4} \cdot 100$$

i.e. Reduction = 90%.

In addittion, if we wish to find out equilibrium temperature of shield:

Let the equilibrium temperature of shield be T_3.

In steady state, we have:

$$Q_{12\_one\_shield} = Q_{13} = Q_{32}$$

Q_{12} is already calculated. Q_{13} or Q_{32} is calculated using Eq. 13.59.

Let us take: $Q_{12\_one\_shield} = Q_{13}$

i.e.

$$Q_{12\_one\_shield} = \frac{A \cdot \sigma \cdot (T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_{31}} - 1}$$

i.e.

$$T_3 := \left[T_1^4 - \frac{Q_{12\_one\_shield} \cdot \left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_{31}} - 1\right)}{A \cdot \sigma} \right]^{\frac{1}{4}}$$

i.e. $T_3 = 979.537$ K

or, $T_3 = 706.537$°C ***(equilibrium temperature of shield.)***

Verify: Use the equation: $Q_{12\_one\_shield} = Q_{32}$

We get, writing for Q_{32}:

$$Q_{12\_one\_shield} = \frac{A \cdot \sigma \cdot (T_3^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_{32}} - 1}$$

i.e.

$$T_3 := \left[T_2^4 - \frac{Q_{12\_one\_shield} \cdot \left(\frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_{32}} - 1\right)}{A \cdot \sigma} \right]^{\frac{1}{4}}$$

i.e. $T_3 = 979.537$ **K** ***(same result as obtained above.)***

Example 13.26. Two very large parallel plates with emissivities 0.3 and 0.7 exchange heat. Find the percentage reduction in heat transfer when two polished aluminium radiation shields ($E = 0.4$) are placed between them. (M.U., Dec. 2000)

Solution. This is the case of two radiation shields placed in between two parallel plates.

Data:

$\varepsilon_1 := 0.3 \qquad \varepsilon_2 := 0.7 \qquad \varepsilon_s := 0.4$

Then, with no radiation shield, we have the radiant heat transfer:

$$Q_{12} = \frac{A \cdot \sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \text{ W}$$ *(for infinitely large parallel plates...(13.59))*

and, with 2 radiation shields, the radiant heat transfer is:

$$Q_{12two\_shields} = \frac{A \cdot \sigma \cdot (T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + 2 \cdot \left(\frac{2}{\varepsilon_s} - 1\right)}$$ *(from Eq. 13.74)*

Therefore, dividing the above two equations, we have:

$$\frac{Q_{12_{two\_shields}}}{Q_{12\_no\_shield}} = \frac{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + 2 \cdot \left(\frac{1}{\varepsilon_s} + \frac{1}{\varepsilon_s} - 1\right)}$$

i.e.
$$\frac{Q_{12_{two\_shields}}}{Q_{12\_no\_shield}} = 0.32$$

i.e. by introducing 2 radiation shields, the heat transfer is reduced to 32% of that without the shields.

Example 13.27. The net radiation from the surface of two parallel plates maintained at temperatures T_1 and T_2 is to be reduced by 79 times. Calculate the number of screens to be placed between two surfaces to achieve this reduction in heat exchange, assuming the emissivity of screens as 0.05 and that of surfaces as 0.8. (M.U)

Solution. This problem is on parallel plates with more than one radiation shields.

Data:

$\varepsilon_1 := 0.8 \qquad \varepsilon_2 := 0.8 \qquad \varepsilon_{s1} := 0.05 \qquad \varepsilon_{s2} := 0.05 \qquad \sigma := 5.67 \times 10^{-8}\ \text{W/(m}^2\text{K)}$ *(Stefan–Boltzmann constant)*

Let N be the number of screens required.

Then, with no radiation shield, we have the radiant heat transfer:

$$Q_{12} = \frac{A \cdot \sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \text{ W}$$ *(for infinitely large parallel plates...(13.59))*

and, with N radiation shields, the radiant heat transfer is:

$$Q_{12N\_shields} = \frac{A \cdot \sigma \cdot (T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + N \cdot \left(\frac{1}{\varepsilon_{s1}} + \frac{1}{\varepsilon_{s2}} - 1\right)}$$ *(from Eq. 13.74)*

Then, by data:
$$\frac{Q_{12\ N\_shields}}{Q_{12}} = \frac{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + N \cdot \left(\frac{1}{\varepsilon_{s1}} + \frac{1}{\varepsilon_{s2}} - 1\right)} = \frac{1}{79} \qquad \text{...(a)}$$

Solving Eq. a, we get N, the number of screens required.

We get:

$$N := \frac{79 \cdot \left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) - \left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)}{\left(\frac{1}{\varepsilon_{s1}} + \frac{1}{\varepsilon_{s2}} - 1\right)}$$

i.e. $N = 3$ *(number of screens required to reduce heat loss by 79 times.)*

Example 13.28. A 10 mm OD pipe carries a cryogenic fluid at 80 K. This pipe is encased by another pipe of 15 mm OD, and the space between the pipes is evacuated. The outer pipe is at 280 K. Emissivities of inner and outer surfaces are 0.2 and 0.3, respectively. (a) Determine the radiant heat flow rate over a pipe length of 5 m. (b) If a radiation shield of diameter 12 mm and emissivity 0.05 on both sides is placed between the pipes, determine the percentage reduction in heat flow. (c) What is the equilibrium temperature of the shield?

Solution.

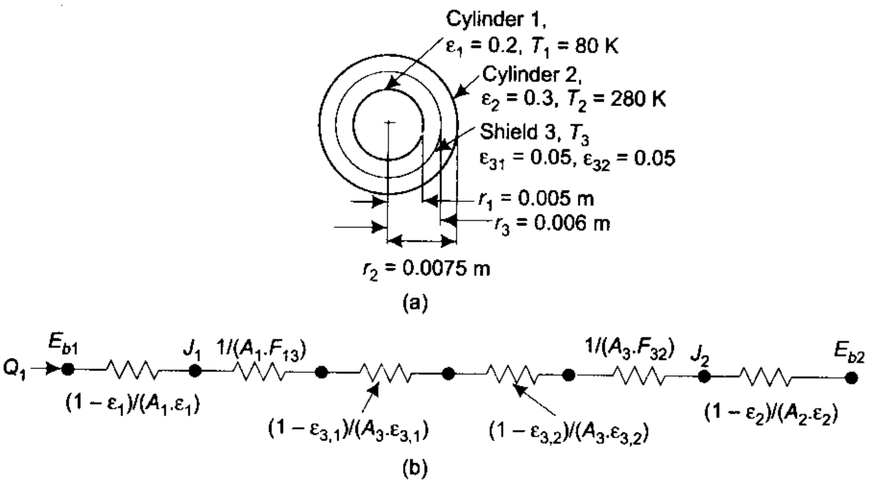


FIGURE Example 13.28 Radiation shield between two concentric cylinders, and associated radiation network

Data:

$r_1 := 0.005$ m $\quad r_2 := 0.0075$ m $\quad r_3 := 0.006$ m $\quad T_1 := 80$ K $\quad T_2 := 280$ K $\quad \varepsilon_1 := 0.2 \quad \varepsilon_2 := 0.3$

$\varepsilon_{31} := 0.05 \quad \varepsilon_{32} := 0.05 \quad \sigma := 5.67 \times 10^{-8}$ W/(m²K) $\quad L := 5$ m

Surface areas for 5 m length:

$A_1 := 2\cdot\pi\cdot r_1\cdot L$ i.e. $A_1 = 0.157$ m² *(surface area of inner pipe)*

$A_2 := 2\cdot\pi\cdot r_2\cdot L$ i.e. $A_2 = 0.236$ m² *(surface area of outer pipe)*

$A_3 := 2\cdot\pi\cdot r_3\cdot L$ i.e. $A_3 = 0.188$ m² *(surface area of radiation shield)*

(a) **Heat transfer without the shield being present:**

We have:

$$Q_{12} := \frac{A_1\cdot\sigma\cdot(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right)\cdot\left(\frac{1}{\varepsilon_2} - 1\right)}$$ *(for infinitely long concentric cylinders...(13.60))*

i.e. $Q_{12} = -8.295$ W

Note that negative sign indicates that heat flow is from outside to inner pipe.

(b) **Heat transfer with one shield being present:**

Now, we have, for heat transfer,

$$Q_{12\_one\_shield} := \frac{A_1\cdot\sigma\cdot(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right)\cdot\left(\frac{1}{\varepsilon_2} - 1\right) + \left(\frac{A_1}{A_3}\right)\cdot\left(\frac{1}{\varepsilon_{31}} + \frac{1}{\varepsilon_{32}} - 1\right)}$$ *(for concentric cylinders with one radiation shield...(13.77))*

i.e. $Q_{12\_one\_shield} = -1.392$ W ***(for concentric cylinders with one radiation shield.)***

Again, note that negative sign indicates that heat flow is from outside to inner pipe.

Therefore percentage reduction in heat flow due to shield:

$$\text{Reduction} := \frac{(8.295 - 1.392)}{8.295}\cdot 100$$

i.e. Reduction = 83.219%.

(c) **Equilibrium temperature of shield:**

Let the equilibrium temperature of shield be T_3.

In steady state, we have:

$$Q_{12\_one\_shield} = Q_{13} = Q_{32}$$

Q_{12} with one shield is already calculated. Q_{13} or Q_{32} is calculated using Eq. 13.59.

Let us take: $Q_{12\_one\_shield} = Q_{13}$

i.e.
$$Q_{12\_one\_shield} = \frac{A_1 \cdot \sigma \cdot (T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_3}\right) \cdot \left(\frac{1}{\varepsilon_{31}} - 1\right)}$$

i.e.
$$T_3 := \left[T_1^4 - \frac{Q_{12\_one\_shield} \cdot \left[\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_3}\right) \cdot \left(\frac{1}{\varepsilon_{31}} - 1\right)\right]}{A_1 \cdot \sigma}\right]^{\frac{1}{4}}$$

i.e. $T_3 = 239.639$ K

or, $T_3 = 33.361$°C *(equilibrium temperature of shield.)*

Verify: Use the equation: $Q_{12\_one\_shield} = Q_{32}$

We get, writing for Q_{32}

$$Q_{12\_one\_shield} = \frac{A_3 \cdot \sigma \cdot (T_3^4 - T_2^4)}{\frac{1}{\varepsilon_{32}} + \left(\frac{A_3}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right)}$$

i.e.
$$T_3 := \left[T_2^4 + \frac{Q_{12\_one\_shield} \cdot \left[\frac{1}{\varepsilon_{32}} + \left(\frac{A_3}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right)\right]}{A_3 \cdot \sigma}\right]^{\frac{1}{4}}$$

i.e. $T_3 = 239.639$ K *(same result as obtained above.)*

Example 13.29. A spherical tank with diameter $D_1 = 40$ cm, filled with a cryogenic fluid at $T_1 = 100$ K, is placed inside a spherical container of diameter $D_2 = 60$ cm, maintained at $T_2 = 300$ K. Emissivities of inner and outer tanks are $\varepsilon_1 = 0.10$ and $\varepsilon_2 = 0.20$, respectively.

(i) Find the rate of heat loss into the inner vessel by radiation

(ii) If a spherical radiation shield of diameter $D_3 = 50$ cm, with an emissivity $\varepsilon_3 = 0.05$ on both surfaces is placed between the spheres, what is the new rate of heat loss? (M.U. Jan. 2002)

Solution. This is a problem on spherical radiation shield. See Fig. Example 13.29 for schematic and the associated radiation network.

Data:

$r_1 := 0.2$ m $r_2 := 0.3$ m $r_3 := 0.25$ m $T_1 := 100$ K $T_2 := 300$ K $\varepsilon_1 := 0.1$ $\varepsilon_2 := 0.2$ $\varepsilon_{31} := 0.05$

$\varepsilon_{32} := 0.05$ $\sigma := 5.67 \times 10^{-8}$ W/(m^{2}K)

Areas:

$A_1 := 4 \cdot \pi \cdot r_1^2$ i.e. $A_1 = 0.503$ m^2

$A_2 := 4 \cdot \pi \cdot r_2^2$ i.e. $A_2 = 1.131$ m^2

and, $A_3 := 4 \cdot \pi \cdot r_3^2$ i.e. $A_3 = 0.785$ m^2

(a) **When there is no radiation shield:**

We have, for radiation heat transfer between two concentric spheres:

$$Q_{12} := \frac{A_1 \cdot \sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right)} \quad \textit{(for concentric spheres...(13.61))}$$

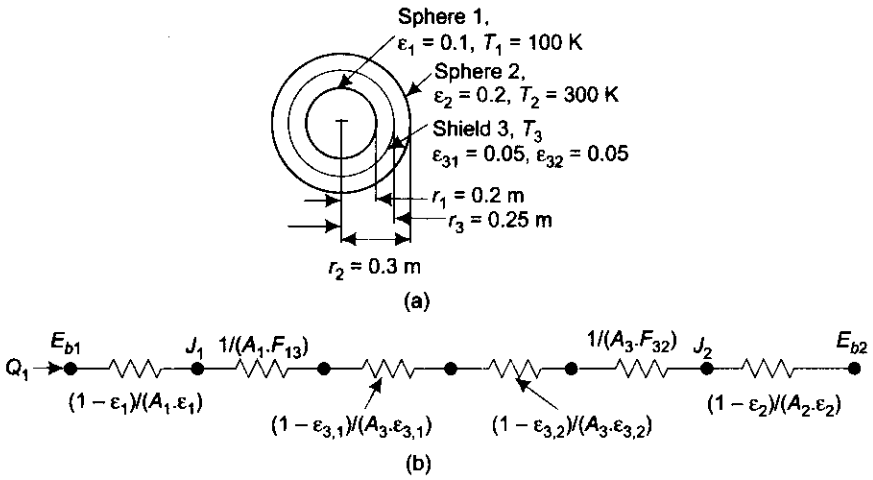


FIGURE Example 13.29 Radiation shield between two concentric spheres, and associated radiation network

i.e. $Q_{12} = -19.359$ W (***radiation heat transfer when there is no shield.***)

Note: Negative sign indicates that heat transfer is radially inwards, i.e. from outer sphere to inner sphere.

(b) When the radiation shield is present:

For radiation heat transfer between concentric spheres with a radiation shield placed in between, we can directly use the Eq. 13.78. However, we shall work from fundamentals, and use the Eq. 13.70, written for the radiation network shown above, and then verify the result from Eq. 13.78:

Now, we have from Eq. (13.70):

$$Q_{12\_one\_shield} = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{A_1 \cdot \varepsilon_1} + \frac{1}{A_1 \cdot F_{13}} + \frac{1-\varepsilon_{31}}{A_3 \cdot \varepsilon_{31}} + \frac{1-\varepsilon_{32}}{A_3 \cdot \varepsilon_{32}} + \frac{1}{A_3 \cdot F_{32}} + \frac{1-\varepsilon_2}{A_2 \cdot \varepsilon_2}}$$

(*for two grey surfaces with one radiation shield placed in between*...(13.70))

Emissive powers:

$E_{b1} := \sigma \cdot T_1^4$ i.e. $E_{b1} = 5.67$ W/m^2 (*Emissive power of surface* 1)

$E_{b2} := \sigma \cdot T_2^4$ i.e. $E_{b2} = 459.27$ W/m^2 (*Emissive power of surface* 2)

View factors:

$F_{13} := 1$ (*since all radiation emitted by surface* 1 *is intercepted by surface* 3)

$F_{32} := 1$ (*since all radiation emitted by surface* 3 *is intercepted by surface* 2)

Resistances:

$R_1 := \frac{1-\varepsilon_1}{A_1 \cdot \varepsilon_1}$ i.e. $R_1 = 17.905$ m^{-2} (*surface resistance of surface* 1)

$R_{13} := \frac{1}{A_1 \cdot F_{13}}$ i.e. $R_{13} = 1.989$ m^{-2} (*space resistance between surfaces* 1 *and* 3)

$R_{3a} := \frac{1-\varepsilon_{31}}{A_3 \cdot \varepsilon_{31}}$ i.e. $R_{3a} = 24.192$ m^{-2} (*surface resistance of* 3, *facing surface* 1)

$R_{3b} := \frac{1-\varepsilon_{32}}{A_3 \cdot \varepsilon_{32}}$ i.e. $R_{3b} = 24.192$ m^{-2} (*surface resistance of* 3, *facing surface* 1)

$R_{32} := \frac{1}{A_3 \cdot F_{32}}$ i.e. $R_{32} = 1.273$ m^{-2} (*space resistance between surfaces* 3 *and* 2)

$R_2 := \frac{1-\varepsilon_2}{A_2 \cdot \varepsilon_2}$ i.e. $R_2 = 3.537$ m^{-2} (*surface resistance of surface* 2)

Then, from Eq. 13.70:

$$Q_{12\_one\_shield} := \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{A_1 \cdot \varepsilon_1} + \frac{1}{A_1 \cdot F_{13}} + \frac{1-\varepsilon_{31}}{A_3 \cdot \varepsilon_{31}} + \frac{1-\varepsilon_{32}}{A_3 \cdot \varepsilon_{32}} + \frac{1}{A_3 \cdot F_{32}} + \frac{1-\varepsilon_2}{A_2 \cdot \varepsilon_2}}$$

i.e. $Q_{12\_one\_shield} = -6.206$ W (***heat transfer between spheres, with radiation shield being present.***)

Note: Negative sign indicates that heat transfer is radially inwards, i.e. from outer sphere to inner sphere.

Verify:

We have, from Eq. 13.78:

$$Q_{12one\_shield} = \frac{A_1 \cdot \sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right) + \left(\frac{A_1}{A_3}\right) \cdot \left(\frac{1}{\varepsilon_{31}} + \frac{1}{\varepsilon_{32}} - 1\right)}$$ *(for concentric spheres with one radiation shield...(13.78))*

where, $\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2$ and, $\frac{A_1}{A_3} = \left(\frac{r_1}{r_3}\right)^2$

i.e. we get:

$$Q_{12\_one\_shield} := \frac{A_1 \cdot \sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \left(\frac{r_1}{r_2}\right)^2 \cdot \left(\frac{1}{\varepsilon_2} - 1\right) + \left(\frac{r_1}{r_3}\right)^2 \cdot \left(\frac{1}{\varepsilon_{31}} + \frac{1}{\varepsilon_{32}} - 1\right)}$$

i.e. $Q_{12\_one\_shield} = -6.206$ **W** (***verified.***)

In addition, if we wish to find out the equilibrium temperature of the shield:

(c) **Equilibrium temperature of shield:**

Let the equilibrium temperature of shield be T_3

In steady state, we have:

$$Q_{12\_one\_shield} = Q_{13} = Q_{32}$$

Q_{12} with one shield is already calculated. Q_{13} or Q_{32} is calculated using Eq. 13.61.

Let us take: $Q_{12\_one\_shield} = Q_{13}$

i.e.

$$Q_{12\_one\_shield} = \frac{A_1 \cdot \sigma \cdot (T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_3}\right) \cdot \left(\frac{1}{\varepsilon_{31}} - 1\right)}$$

i.e.

$$T_3 := \left[T_1^4 - \frac{Q_{12\_one\_shield} \cdot \left[\frac{1}{\varepsilon_1} + \left(\frac{A_1}{A_3}\right) \cdot \left(\frac{1}{\varepsilon_{31}} - 1\right)\right]}{A_1 \cdot \sigma}\right]^{\frac{1}{4}}$$

i.e. $T_3 = 264.919$ K

or, $T_3 = 8.081$°C (***equilibrium temperature of shield.***)

Verify Use the equation: $Q_{12\_one\_shield} = Q_{32}$

We get, writing for Q_{32}

$$Q_{12\_one\_shield} = \frac{A_3 \cdot \sigma \cdot (T_3^4 - T_2^4)}{\frac{1}{\varepsilon_{32}} + \left(\frac{A_3}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right)}$$

i.e.

$$T_3 := \left[T_2^4 + \frac{Q_{12\_one\_shield} \cdot \left[\frac{1}{\varepsilon_{32}} + \left(\frac{A_3}{A_2}\right) \cdot \left(\frac{1}{\varepsilon_2} - 1\right)\right]}{A_3 \cdot \sigma}\right]^{\frac{1}{4}}$$

i.e. $T_3 = 264.919$ K *(same result as obtained above.)*

13.9 Radiation Error in Temperature Measurement

An important application of radiation shields is in reducing the radiation error in temperature measurement. To explain this, consider the case of a hot fluid at a temperature T_f, flowing through a channel, whose walls are at a temperature T_w. Let the convective heat transfer coefficient between the fluid and the thermometer bulb be h. To measure the temperature of the fluid, a thermometer (or a thermocouple) is introduced into the stream, as shown in Fig. 13.39 (a).

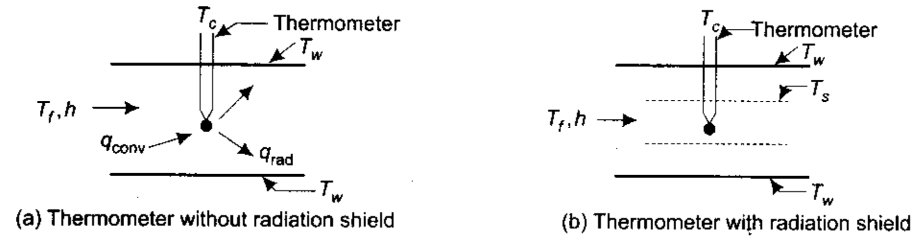


FIGURE 13.39 Radiation shielding of thermometers

Let the reading shown by the thermometer be T_c. This reading, however, does not represent the true temperature of the fluid T_f, since the thermometer bulb will lose heat by radiation to the walls of the channel which are at a lower temperature T_w (which is usually the case). So, in steady state, the thermometer bulb will gain heat by convection from the flowing fluid and will lose heat by radiation to the walls, and as a result, the temperature T_c shown by the thermometer will be some value in between T_f and T_w.

We wish to find out the true temperature of the fluid T_f, by knowing the thermometer reading T_c.

Making an energy balance on the thermometer bulb, in steady state, we have:

Without radiation shield:

$$q_{conv} \text{ to the bulb} = q_{rad} \text{ from the bulb}$$

i.e. $$h \cdot A_c \cdot (T_f - T_c) = \varepsilon_c \cdot A_c \cdot \sigma \cdot (T_c^4 - T_w^4)$$

i.e. $$T_f = T_c + \frac{\varepsilon_c \cdot \sigma \cdot (T_c^4 - T_w^4)}{h} \qquad ...(13.79)$$

where,

A_c = surface area of thermometer bulb,

ε_c = emissivity of thermometer bulb surface.

Eq. 13.79 gives the true temperature of the fluid T_f. Second term on the RHS of Eq. 13.79 represents the error in temperature measurement due to radiation effect. It is clear that radiation error can be minimised by:

(i) having low value of ε_c, i.e. high reflectivity for the bulb surface

(ii) high value for convective heat transfer coefficient, h.

In practice, even if we start with a thermometer bulb surface of high reflectivity, soon, the emissivity value rises to about 0.8 or 0.9 due to deposit formation, corrosion or erosion of the bulb surface, etc.

So, the most practical way to reduce the radiation error in temperature measurement is to provide a cylindrical radiation shield around the thermometer bulb, as shown in Fig. 13.39 (b). Then, in steady state, the shield temperature (T_s) will stabilise somewhere in between the fluid temperature T_f and the wall temperature T_w. Then, in Eq. 13.79, T_w will be replaced by the effective shield temperature T_s.

Energy balance on the thermometer bulb:

Heat transferred to the bulb from the fluid by convection = Heat transferred from the bulb to the shield by radiation,

i.e. $$h \cdot A_c \cdot (T_f - T_c) = \frac{\sigma \cdot (T_c^4 - T_s^4)}{\left(\frac{1-\varepsilon_c}{A_c \cdot \varepsilon_c}\right) + \frac{1}{A_c \cdot F_{cs}} + \left(\frac{1-\varepsilon_s}{A_s \cdot \varepsilon_s}\right)} \qquad ...(13.80)$$

In Eq. 13.80, F_{cs} = view factor of thermometer bulb w.r.t. the shield and is, generally equal to 1.

In the RHS of eqn. (13.80), first term in the denominator represents surface resistance of the bulb, second term is the space resistance between the bulb and the shield and the third term is the surface resistance of the shield.

Now, making an energy balance on the shield:

(heat transferred to shield from the fluid by convection + heat transferred to shield from bulb by radiation) = heat transferred from shield to walls by radiation

i.e.
$$2 \cdot A_s \cdot h \cdot (T_f - T_s) + \frac{\sigma \cdot (T_C^4 - T_S^4)}{\left(\frac{1-\varepsilon_C}{A_C \cdot \varepsilon_C}\right) + \frac{1}{A_C \cdot F_{CS}} + \left(\frac{1-\varepsilon_S}{A_S \cdot \varepsilon_S}\right)} = \varepsilon_s \cdot A_s \cdot \sigma \cdot (T_s^4 - T_w^4) \quad ...(13.81)$$

where, A_s = area of shield on one side

ε_s = emissivity of shield surface

A_c = area of bulb surface

ε_c = emissivity of bulb surface

F_{cs} = view factor of bulb w.r.t. shield.

In the first term of the above equation factor 2 appears since convective heat transfer to the shield occurs on both surfaces of the shield. Also, in writing the RHS, the inherent assumption is that:

$F_{sw} = 1$ *(view factor between the shield and the walls)*

and,

$\frac{A_s}{A_w} = 0$ *(i.e. surface area of shield is negligible compared to the area of the channel walls.)*

Solving Eqs. 13.80 and 13.81 simultaneously, we obtain the shield temperature T_s and the thermometer reading T_c, (if T_f is known), or T_f (if T_c is known).

Example 13.30. Hot air is flowing in a duct whose walls are maintained at a temperature T_w = 450 K. A thermocouple placed in the stream shows a reading of 650 K. If the emissivity of the thermocouple junction is ε_c = 0.8 and the convective heat transfer coefficient between the flowing air and the thermocouple is h = 85 W/(m^{2}C), find out the true temperature of the flowing stream.

(b) Now, if a radiation shield (ε_s = 0.3) is placed between the thermocouple and the walls, what will be new value of T_c read by the thermocouple? And, how much is the temperature error? Take $A_c/A_s = 1/5$.

Solution. In case (a), there is no radiation shield and in case (b), the radiation shield is present. Both these cases are shown in Fig. Example 13.30 (a) and (b).

Data:

T_w := 450 K $\quad T_c$:= 650 K $\quad h$:= 85 W/(m^{2}C) $\quad \varepsilon_c$:= 0.8 $\quad \varepsilon_s$:= 0.3 $\quad \frac{A_c}{A_s} = \frac{1}{5}$

σ := 5.67 × 10^{-8} W/(m^{2}K) *(Stefan–Boltzmann constant.)*

Case (a): When there is no radiation shield:

In steady state, making a heat balance on the thermocouple bead, we have:

$$q_{conv} = q_{rad}$$

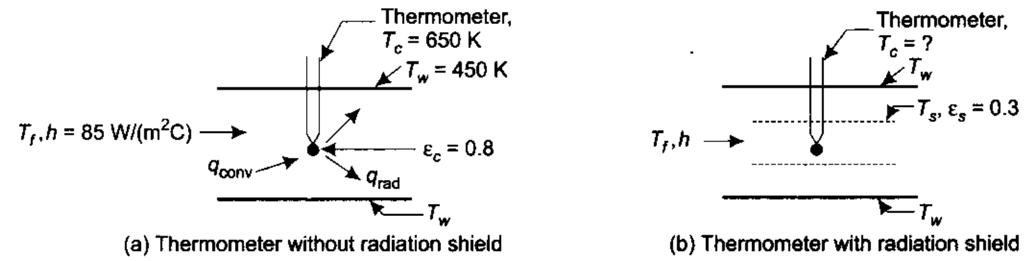


FIGURE Example 13.30 Radiation shielding of thermometers i.e. $$h \cdot A_c \cdot (T_f - T_c) = \sigma \cdot \varepsilon_c \cdot A_c \cdot (T_c^4 - T_w^4)$$

i.e. $$T_f := T_c + \frac{\varepsilon_c \cdot \sigma \cdot (T_c^4 - T_w^4)}{h} \quad ...(13.79)$$

i.e. $T_f = 723.376$ K (***true temperature of air stream, when there is no radiation shield***

Therefore, radiation error $= T_f - T_c = 73.376$ **deg.**

Case (b): When the radiation shield is present:

Making a heat balance on the thermocouple bead:

$$h \cdot A_c \cdot (T_f - T_c) = \frac{\sigma \cdot (T_c^4 - T_s^4)}{\left(\frac{1-\varepsilon_c}{A_c \cdot \varepsilon_c}\right) + \frac{1}{A_c \cdot F_{cs}} + \left(\frac{1-\varepsilon_s}{A_s \cdot \varepsilon_s}\right)} \quad ...(13.80)$$

where, $F_{cs} = 1$ (*view factor for thermocouple bead w.r.t. shield.*)

Then, Eq. 13.80 becomes:

$$h \cdot A_c \cdot (T_f - T_c) = \frac{\sigma \cdot A_c \cdot (T_c^4 - T_s^4)}{\frac{1}{\varepsilon_c} + \left(\frac{A_c}{A_s}\right) \cdot \left(\frac{1}{\varepsilon_s} - 1\right)}$$

i.e. $$h \cdot A_c \cdot (T_f - T_c) = \frac{\sigma \cdot A_c \cdot (T_c^4 - T_s^4)}{\frac{1}{\varepsilon_c} + \left(\frac{1}{5}\right) \cdot \left(\frac{1}{\varepsilon_s} - 1\right)} \quad (a)$$

Next, making a heat balance on the shield:

$$2 \cdot A_s \cdot h \cdot (T_f - T_s) + \frac{\sigma \cdot (T_c^4 - T_s^4)}{\left(\frac{1-\varepsilon_c}{A_c \cdot \varepsilon_c}\right) + \frac{1}{A_c \cdot F_{cs}} + \left(\frac{1-\varepsilon_s}{A_s \cdot \varepsilon_s}\right)} = \varepsilon_s \cdot A_s \cdot \sigma \cdot (T_s^4 - T_w^4) \quad ...(13.81)$$

where,

A_s = area of shield on one side
ε_s = emissivity of shield surface
A_c = area of bulb surface
ε_c = emissivity of bulb surface
F_{cs} = view factor of bulb w.r.t. shield = 1
$F_{sw} = 1$ (*view factor between the shield and the walls*)

and,

$\frac{A_s}{A_w} = 0$ (*i.e. surface area of shield is negligible compared to the area of the channel walls*)

Then, Eq. 13.81 becomes:

$$2 \cdot A_s \cdot h \cdot (T_f - T_s) + \frac{\sigma \cdot A_c \cdot (T_c^4 - T_s^4)}{\frac{1}{\varepsilon_c} + \left(\frac{A_c}{A_s}\right) \cdot \left(\frac{1}{\varepsilon_s} - 1\right)} = \varepsilon_s \cdot A_s \cdot \sigma \cdot (T_s^4 - T_w^4)$$

i.e. $$2 \cdot \left(\frac{A_s}{A_c}\right) \cdot h \cdot (T_f - T_s) + \frac{\sigma \cdot (T_c^4 - T_s^4)}{\frac{1}{\varepsilon_c} + \left(\frac{A_c}{A_s}\right) \cdot \left(\frac{1}{\varepsilon_s} - 1\right)} = \varepsilon_s \cdot \left(\frac{A_s}{A_c}\right) \cdot \sigma \cdot (T_s^4 - T_w^4)$$

i.e. $$10 \cdot h \cdot (T_f - T_s) + \frac{\sigma \cdot (T_c^4 - T_s^4)}{\frac{1}{\varepsilon_c} + \left(\frac{1}{5}\right) \cdot \left(\frac{1}{\varepsilon_s} - 1\right)} = \varepsilon_s \cdot (5) \cdot \sigma \cdot (T_s^4 - T_w^4), \text{ since } \frac{A_c}{A_s} = \frac{1}{5} \quad (b)$$

Now, T_f is already known, and solving Eqs. a and b simultaneously, we get T_c and T_s.

To do this, we use solve block of Mathcad. We start with trial values of T_c and T_s, and write the constraint Eqs. a and b immediately after typing 'Given'. Then, typing 'Find' (T_c, T_s) = gives immediately the values of T_c and T_s.

$T_c := 100 \quad T_s := 100$ *(trial values of T_c and T_s)*

Given

$$h\cdot(T_f - T_c) = \frac{\sigma\cdot(T_c^4 - T_s^4)}{\frac{1}{\varepsilon_c} + \left(\frac{1}{5}\right)\cdot\left(\frac{1}{\varepsilon_s} - 1\right)} \qquad \text{(a)}$$

$$10\cdot h\cdot(T_f - T_s) + \frac{\sigma\cdot(T_c^4 - T_s^4)}{\frac{1}{\varepsilon_c} + \left(\frac{1}{5}\right)\cdot\left(\frac{1}{\varepsilon_s} - 1\right)} = \varepsilon_s\cdot(5)\cdot\sigma\cdot(T_s^4 - T_w^4) \qquad \text{(b)}$$

$$\text{Find}\,(T_c, T_s) = \begin{bmatrix} 716.327 \\ 703.655 \end{bmatrix}$$

i.e. $T_c = 716.327$ K *(value of thermocouple reading, when the shield is present.)*

And, $T_s = 703.655$ K *(temperature of shield.)*

Therefore, radiation error $= T_f - T_c =$ **7.049 deg.**

Note: When there is no radiation shield, the error in thermocouple reading is 73.376 deg. and when the radiation shield is introduced, the radiation error is reduced to just 7.049 deg.

13.10 Radiation Heat Transfer Coefficient (h_r)

Concept of "radiation heat transfer coefficient" is useful in solving problems where heat transfer occurs by both convection and radiation. Typical examples of such a situation are: heat loss from a steam pipe passing through a room, heat loss from hot combustion products passing through a duct, heat loss from the walls and door of a furnace, etc.

Radiation heat transfer coefficient is defined in a manner analogous to convection heat transfer coefficient. Consider hot gases at a temperature T_g flowing through a tube whose walls are at a temperature of T_w. Then, recollect that the convective heat flux is given by:

$$q_{conv} = h_c\cdot(T_g - T_w)$$

where, h_c = convective heat transfer coefficient.

In a similar manner, we write for radiant heat flux from the pipe:

$$q_{rad} = h_r\cdot(T_g - T_w)$$

where, h_r = radiation heat transfer coefficient.

For the above case, h_r is determined from:

$$h_r\cdot(T_g - T_w) = \varepsilon\cdot\sigma\cdot(T_g^4 - T_w^4)$$

i.e.

$$h_r = \frac{\varepsilon\cdot\sigma\cdot(T_g^4 - T_w^4)}{(T_g - T_w)}$$

i.e.

$$h_r = \varepsilon\cdot\sigma\cdot(T_g^2 + T_w^2)\,(T_g + T_w)\ \text{W/(m}^2\text{C)} \qquad \text{...(13.82)}$$

Then, $q_{tot} = q_{conv} + q_{rad}$, by a linear superposition of both heat fluxes.

$$= h_c\cdot(T_g - T_w) + h_r\cdot(T_g - T_w)$$

$$= (h_c + h_r)\cdot(T_g - T_w) \qquad \text{...(13.83)}$$

For any other configuration, we can determine h_r if we know the expressin for radiant heat flux. For example, for radiant heat transfer between two large parallel plates, we have:

$$\frac{Q}{A} = \frac{\sigma\cdot(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = h_r\cdot(T_1 - T_2)$$

i.e.

$$h_r = \frac{\sigma\cdot(T_1^2 + T_2^2)\cdot(T_1 + T_2)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}\ \text{W/(m}^2\text{C)} \qquad \textit{(for two parallel plates...(13.84))}$$

Note that radiation heat transfer coefficient is a strong function of temperature, unlike the convective heat transfer coefficient.

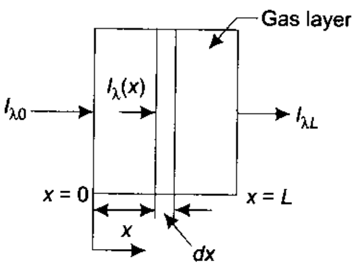


FIGURE 13.40 Absorption of monochromatic radiation in a gas layer

13.11 Radiation from Gases, Vapours and Flames

So far, we have dealt with radiation heat exchange between surfaces in an enclosure, with a non-participating medium, in between, i.e. the intervening gas neither absorbs nor scatters the radiation nor does it emit any radiation; in other words, the intervening gas does not, in any way, affect the radiant heat transfer between the surfaces. Such an assumption is valid for mono-atomic gases such as argon and helium, and for diatomic gases such as oxygen and nitrogen; these gases are extremely inert to thermal radiation. However, the same thing is not true for poly-atomic gases such as CO_2, H_2O (vap), NH_3 and hydrocarbon gases; these gases do absorb and emit radiation. Further, radiation from solids and liquids generally covers the entire wavelength range whereas radiation from gases is over selected wavelength 'bands'. Also, note that radiation from solids is a 'surface phenomenon' whereas that from gases is a 'volumetric phenomenon'.

13.11.1 Volumetric Absorption and Emissivity

In gases, absorption of radiation depends upon the absorption coefficient κ_λ (1/m) and thickness L of the gas layer, in addition to the temperature T_g of the gas. Fig. 13.40 shows a monochromatic beam of intensity $I_{\lambda 0}$ impinging on the gas layer at $x = 0$; its intensity decreases as a result of absorption and at $x = L$, let the intensity be $I_{\lambda L}$.

We wish to develop a relation between the initial and final intensities: If I_λ is the intensity at any x, the reduction in intensity occurring in an infinitesimal layer of thickness dx is given by:

$$dI_\lambda(x) = -\kappa_\lambda \cdot I_\lambda(x) \cdot dx$$

Separating the variables and integrating from 0 to L, i.e. over the entire thickness of gas layer, we get:

$$\int_{I_{\lambda 0}}^{I_{\lambda L}} \frac{1}{I_\lambda(x)} dI_\lambda(x) = -k_\lambda \int_0^L 1 dx$$

where, the absorption coefficient κ_λ is assumed to be independent of x.

We get:

$$\frac{I_{\lambda L}}{I_{\lambda 0}} = \exp(-\kappa_\lambda \cdot L) \quad ...(13.85)$$

This is known as **Beer's Law**.

i.e. the intensity of radiation decreases exponentially with thickness as it travels through the gas layer.

LHS of Eq. 13.85 is monochromatic transmissivity τ_λ of the gas. Also, in general, gases do not reflect radiation, i.e. their reflectivity is zero. Therefore, we write:

$$\alpha_\lambda + \tau_\lambda = 1$$

i.e. $$\alpha_\lambda = 1 - \tau_\lambda$$

i.e. $$\alpha_\lambda = 1 - \exp(-\kappa_\lambda \cdot L)$$ (*monochromatic absorptivity of gas...(13.86)*)

Then, from Kirchhoff's law, since absorptivity is equal to emissivity, we have:

i.e. $$\varepsilon_\lambda = 1 - \exp(-\kappa_\lambda \cdot L)$$ (*spectral emissivity of gas...(13.87)*)

From Eq. 13.87, one can see that if gas layer thickness, L is very large,

$$\alpha_\lambda = \varepsilon_\lambda = 1$$

i.e. for very thick layers, radiation from the gas is equivalent to a black body radiation.

13.11.2 Gaseous Emission and Absorption

As mentioned earlier, gases are 'selective' absorbers and emitters, i.e. gases absorb or emit radiant energy only within certain wavelength bands. Beyond these wavelength bands, these gases are transparent (or diathermic) to thermal radiation. In thermal engineering, we are particularly interested in CO_2 and H_2O vapour, since these are the main products of combustion of fuels.

Following wavelength bands are of importance for CO_2 and H_2O vapour:

For CO_2:

Band 1: λ = 2.40 to 3.80 microns

Band 2: λ = 4.01 to 4.80 microns

Band 3: λ = 12.5 to 16.5 microns

For H_2O vapour:

Band 1: λ = 2.24 to 3.27 microns

Band 2: λ = 4.80 to 8.5 microns

Band 3: λ = 12.0 to 25 microns.

As discussed earlier, intensity of radiation decreases as it passes through a gas layer; this 'attenuation' in intensity is proportional to the path length 'L' and the partial pressure 'p' of the gas (in a mixture of gases). Emissive power of a gas is proportional to the gas temperature T_g and the product ($p.L$). From the experimental data for CO_2 and H_2O, following empirical relations for the emissive powers of CO_2 and H_2O have been suggested:

$$E_{CO_2} = 3.5 \cdot (p \cdot L)^{\frac{1}{3}} \cdot \left(\frac{T}{100}\right)^{3.5} \text{ kcal/(m}^2\text{hr.)} \qquad \text{...(13.88)}$$

$$E_{H_2O} = 3.5 \cdot p^{0.8} \cdot L^{0.6} \cdot \left(\frac{T}{100}\right)^{3.0} \text{ kcal/(m}^2\text{hr.)} \qquad \text{...(1389)}$$

where, p = partial pressure (atm) and L = layer thickness (m).

(**Note:** 1 kcal/(m^{2}hr) = 1.162 × 10^{-3} kW/m^2).

For a diffuse surface, radiation is emitted in all directions. Therefore, path length L depends on direction and shape of the body. For calculation purposes, a 'mean path length of beam' (L) is defined as follows:

$$L = 3.6 \cdot \left(\frac{V}{A}\right) \text{ m} \qquad \textit{(mean path length...(13.90))}$$

where, V is the volue of the body (m^3), and A is the surface area of enclosure (m^2).

Emissivity of gases is a function of gas temperature T_g, total pressure p of the gas mixture, partial pressure p_g of the radiating gas and the mean path length, L.

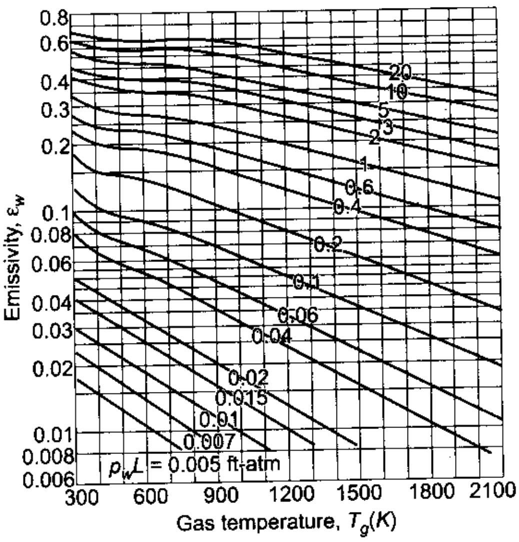


FIGURE 13.41 Emissivity of water vapour in a mixture of other gases which are non-radiating, at a total pressure of 1 atm. (Source: Incropera, Frank P. and David P. Dewitt [1998]. *Fundamentals of Heat and Mass Transfer.* Pub.: John Wiley & Sons)

Emissivity of water vapour (ε_w) in a mixture of other gases which are non-radiating, at a total pressure of 1 atm. are plotted, as a function of gas temperature T_g and the product of partial pressure of water vapour and the mean path length, ($p_w.L$), in Fig. 13.41:

To determine emissivity of water vapour when the total pressure is different from one atm., multiply the value obtained from Fig. 13.41 by a correction factor (C_w), obtained from Fig. 13.42:

Similarly, Fig. 13.43 shows a plot of emissivity of carbon dioxide gas in a mixture of other gases which are non-radiating, at a total pressure of 1 atm. and the Fig. 13.44 shows correction factor C_c for emissivity of carbon dioxide, when the total pressure is other than 1 atm.

When water vapour and carbon dioxide appear together in a mixture of other non-radiating gases, *total* gas emissivity (ε_g) is expressed as:

$$\varepsilon_g = \varepsilon_w + \varepsilon_c - \Delta\varepsilon \qquad \text{...(13.91)}$$

In Eq. 13.91, $\Delta\varepsilon$ is the correction factor, read from Fig. 13.45. Note that total emissivity is less than the sum of the individual emissivities of water vapour and carbon dioxide because of mutual absorption of radiation between these two gases.

Mean path length (L) to be used in Figs. 13.41 to 13.45, for various geometries, are given in Table 13.6:

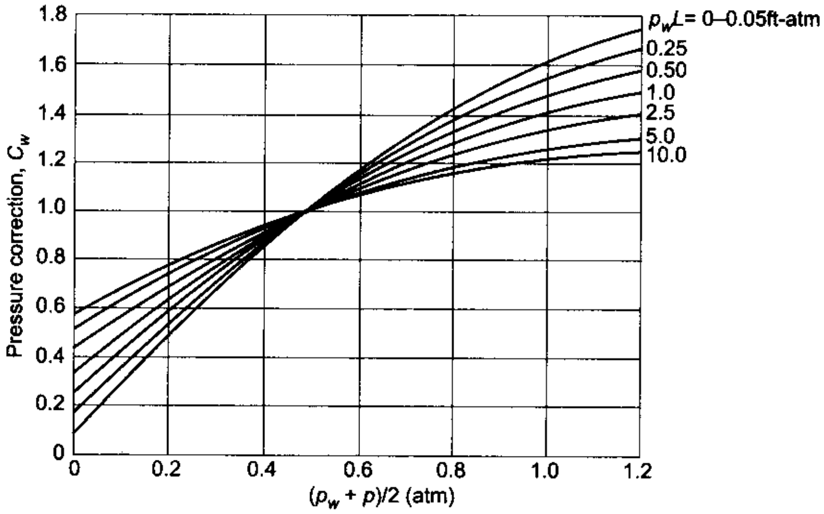


FIGURE 13.42 Correction factor for emissivity of water vapour when the total pressure of mixture is other than 1 atm. (Source: Incropera and Dewitt [1998]. op. cit.)

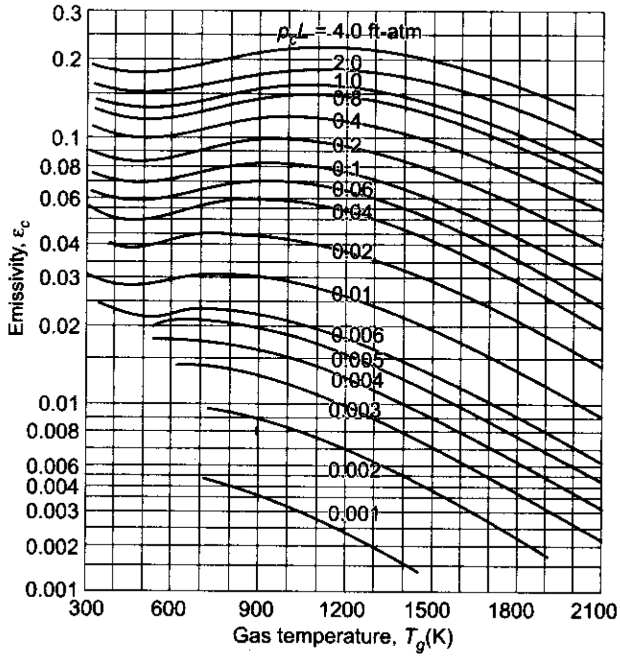


FIGURE 13.43 Emissivity of carbon dioxide in a mixture of other gases which are non-radiating, at a total pressure of 1 atm. (Source: Incropera and Dewitt [1998]. op. cit.)

Once the emissivity (ε_g) of the gas mass in the given geometry is determined, we can proceed to find out the radiant heat transfer from the gas mass to the surface of enclosure:

If the surface is black: Radiation emitted by the gas mass is completely absorbed by the black surface; black surface also emits radiation which, in turn, is absorbed by the gas depending upon its absorptivity. Therefore, the net radiant heat exchange between the gas mass at a temperature T_g and the surface at a temperature T_s is:

$$Q_{net} = A_s \cdot \sigma \cdot (\varepsilon_g \cdot T_g^4 - \alpha_g \cdot T_s^4) \qquad ...(13.92)$$

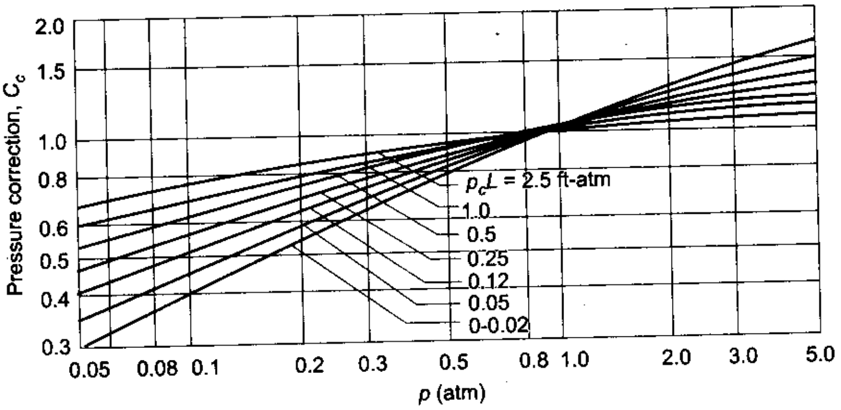


FIGURE 13.44 Correction factor for emissivity of carbon dioxide when the total pressure of mixture is other than 1 atm. (Source: Incropera and Dewitt [1998]. op. cit.)

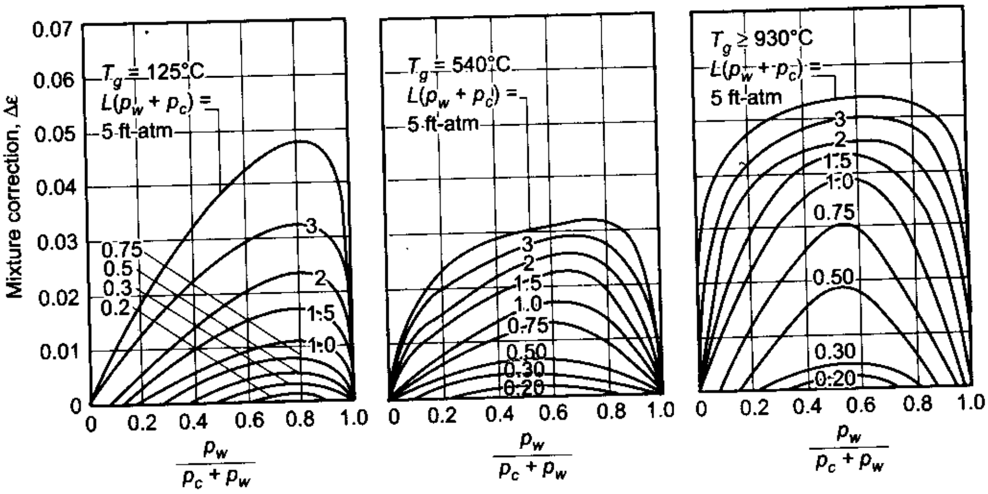


FIGURE 13.45 Correction factor for mixtures of carbon dioxide and water vapour. (Source: Incropera and Dewitt [year]. op. cit.)

The absorptivity, α_g for water vapour and carbon dioxide is calculated as follows:

For water vapour:

$$\alpha_w = C_w \cdot \left(\frac{T_g}{T_s}\right)^{0.45} \cdot \varepsilon_w\left(T_s, \frac{p_w \cdot L \cdot T_s}{T_g}\right) \quad \text{...(13.93)}$$

For carbon dioxide:

$$\alpha_c = C_c \cdot \left(\frac{T_g}{T_s}\right)^{0.65} \cdot \varepsilon_c\left(T_s, \frac{p_c \cdot L \cdot T_s}{T_g}\right) \quad \text{...(13.94)}$$

Correction factors C_w and C_c are obtained from Figs. 13.42 and 13.44, respectively. Emissivities ε_w and ε_c are obtained from Figs. 13.41 and 13.43, respectively, however, replacing T_g by T_s in the x-axis, and replacing $(p_w.L)$ or $(p_c.L)$ by $\{p_w.L.(T_s/T_g)\}$ or $\{p_c.L.(T_s/T_g)\}$, respectively.

When both water vapour and carbon dioxide are present in gas mixture, total gas absorptivity, (α_g) is obtained as:

TABLE 13.6 Mean beam lengths for various gas geometries

Geometry	Characteristic dimension	Mean path length (L)
Cylinder (height = diameter), radiating to whole surface	Diameter, D	0.60 D
Cylinder (height = diameter), radiating to centre of base	Diameter, D	0.71 D
Cylinder (height = 0.5 diameter), radiating to:	Diameter, D	
(a) end		0.43 D
(b) side		0.46 D
(c) whole surface		0.45 D
Sphere, radiating to entire surface	Diameter, D	0.65 D
Hemisphere, radiating to element in centre of base	Radius, R	R
Cube, radiating to any face	Edge, L	0.60 L
Two infinite planes	Separation distance, L	1.8 L
Bank of tubes, diameter = D, distance between surfaces of tubes = x:		
(a) triangular arrangement, x = D		(2.8) x
(b) triangular arrangement, x = 2D		(3.8) x
(c) square arrangement, x = D		(3.5) x
Arbitrary shape of volume, V (radiation to surface of area, A)	Volume to area ratio, (V/A)	3.6(V/A)

$$\alpha_g = \alpha_w + \alpha_c - \Delta\alpha \quad \text{...(13.95)}$$

where, $\Delta\alpha = \Delta\varepsilon$ is obtained from Fig. 13.45.

If the surface is grey: This is the most probable case, since with passage of time, enclosure walls will get dirty, and the surface emissivity ε_s becomes less than unity. However, effective emissivity of the surface $\varepsilon_{s\text{-eff}}$ in the presence of gas mass is greater than ε_s ; for ε_s = 0.8 to 1.0, we have the approximate formula for $\varepsilon_{s\text{-eff}}$:

$$\varepsilon_{s\_eff} = \frac{(\varepsilon_s + 1)}{2} \quad \text{...(13.92)}$$

Then, the net radiant heat exchange between the gas mass at a temperature T_g and the surface at a temperature T_s is given by:

$$Q_{net} = \varepsilon_{s\_eff} \cdot A_s \cdot \sigma \cdot (\varepsilon_g \cdot T_g^4 - \alpha_g \cdot T_s^4) \quad \text{...(13.93)}$$

Radiation from flames Flame is produced during combustion (of a fuel). Radiation from flames occurs in furnaces, jet engine burners, etc. Flames may be luminous or non-luminous. Flames produced by household stoves (burning kerosene or wood) are not luminous. Luminous flames have glowing particles of carbon, soot and flying ash, and involve high temperatures. Radiation from the flame, obviously, depends on the emission of particles contained in the flame, which in turn, depends on the kind of fuel burnt, mode of combustion, design of the furnace, amount of air introduced, etc. Net radiation heat exchange between a flame and its enclosure is given by:

$$Q_{net} = \sigma \cdot A_f \cdot F_{fw} \cdot \varepsilon_f \cdot \varepsilon_w \cdot (T_f^4 - T_w^4) \quad \text{...(13.94)}$$

where, A_f is the area of the flame envelope, subscripts '*f*' and '*w*' refer to the flame and wall surface, respectively.

'Effective flame temperature, T_f,' (in Kelvin) is generally calculated as the geometric mean of the theoretical temperature of combustion T_1 and the temperature of combustion products, T_2, at the furnace outlet.

i.e. $$T_f = \left(T_1^2 \cdot T_2^2\right)^{\frac{1}{4}} \text{ K} \quad \text{...(13.95)}$$

Approximate values of flame emissivity (ε_f) for flames of different fuels are given in Table 13.7:

Example 13.31. A spherical chamber of 0.8 m diameter is filled with a gas mixture at 1 atm. and is at 1500 K. The gas mixture contains 20% CO_2 by volume, and the rest of the mixture is non-radiating gases. Determine the emissivity of the gas body.

TABLE 13.7 Flame emissivity (ε_f) for an infinitely thick layer

Kind of flame	*Flame emissivity, ε_f*
Non-luminous gas flame (or, anthracite in grate stoker combustion)	0.40
Luminous flame of pulverised anthracite	0.45
Luminous flame of lean coal	0.60
Luminóus flame of coal with large volatile content (brown coal, peat, etc., burned in a layer or pulverised)	0.70
Luminous masut flame	0.85

(b) If the volume is filled to a pressure of 3 atm., but with the fraction of CO_2 still being 20%, what will be the value of emissivity of gas body?

Solution. This is a spherical gas body. From Table 13.6, we see that for a spherical body, the mean path length of beam is 0.65 D, where D is the diameter of sphere.

$D := 0.8$ m $\quad T_g := 1500$ K $\quad L := 0.65$ D i.e. $L = 0.52$ m $\quad p := 1$ atm. $\quad p_c := 0.2$ atm.

Therefore, $\quad p_c \cdot L = 0.104$ m. atm.

i.e. $\quad p_c \cdot L = 0.104 \times 3.28$ ft. atm.

i.e. $\quad p_c \cdot L = 0.341$ ft. atm.

Now, refer to Fig. 13.43. For $T_g = 1500$ K, and $p_c \cdot L = 0.341$ ft. atm., we read:

$\varepsilon_c = 0.09$ *(emissivity of carbon dioxide = emissivity of gas mixture)*

(b) **When the total pressure is 3 atm., with volume fraction of CO_2 being 20%:**

Now, L remains the same, but p_c will be:

$p_c = 0.2 \times 3$ atm.

i.e. $\quad p_c = 0.6$ atm.

Then, $\quad p_c \cdot L = 0.312$ m. atm.

i.e. $\quad p_c \cdot L = 0.312 \times 3.28$ ft. atm.

i.e. $\quad p_c \cdot L = 1.023$ ft. atm.

Now, refer to Fig. 13.43. For $T_g = 1500$ K, and $p_c \cdot L = 1.023$ ft. atm., we read:

$\varepsilon_c = 0.14$ *(emissivity of carbon dioxide.)*

However, this value of emissivity is for a total pressure of 1 atm. In the present case, total pressure is 3 atm. Therefore, obtained value of 0.14 has to be multiplied by a correction factor, read from Fig. 13.44. We get, from Fig. 13.44, for total pressure, $p = 3$ atm. And, $p_c \cdot L = 1.023$ ft. atm.,

$C_c = 1.35$ *(correction factor)*

Therefore, emissivity of CO_2 when the mixture pressure is 3 atm.:

$\varepsilon_c = 0.14 \times 1.35$

i.e. $\quad \varepsilon_c = 0.189$ *(emissivity of CO_2 when the mixture pressure is 3 atm.)*

13.12 Solar and Atmospheric Radiation

We give a brief introduction to this fascinating topic because of its importance in the context of the 'energy crisis' and the resulting interest in 'renewable energy sources'; further, this is a topic which affects our daily life.

Energy emitted by the sun is known as 'solar energy'. Inexhaustible energy of sun is produced as a result of nuclear 'fusion' reaction between two hydrogen atoms to form one atom of helium. 'Atmospheric radiation' is the radiation emitted or reflected by the constituents of the atmosphere.

Sun is a spherical body of diameter $D = 1.39 \times 10^9$ m and is located at a mean distance of $L = 1.50 \times 10^{11}$ m from the earth. Even though the sun radiates an enormous amount of energy, only less than a billionth of this energy reaches the earth's surface. Solar radiation travels through the vacuum of space till it encounters earth's atmosphere. By conducting experiments with high altitude aircraft or balloons, and spacecrafts, scientists have shown that average value of solar energy reaching the upper surface of earth's atmosphere is about 1353 W/m^2. This value is known as **solar constant, G_s**. The solar constant is the rate at which solar energy is incident on a surface normal to the sun's rays at the outer edge of the atmosphere when the earth is at its mean distance from the sun. Since the earth moves in an elliptical orbit around the sun, this mean distance (L) varies with the

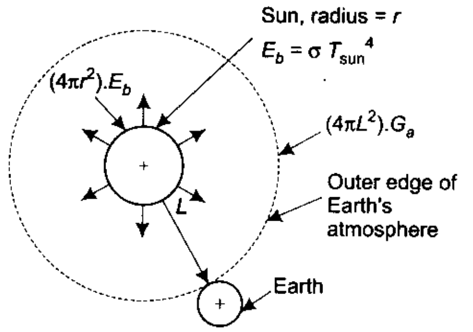


FIGURE 13.46 Estimation of surface temperature of sun when the solar constant is known

position of the earth and the value of G_s also varies; however, the average value of G_s taken is 1353 W/m². Constituents of the atmosphere absorb and/or scatter radiations of different wavelengths contained in solar radiation. As a result, the amount of solar energy actually reaching the earth's surface is about 950 W/m².

From the measured value of solar constant, we can easily determine the surface temperature of the sun. See Fig. 13.46.

We use the condition that total energy radiated by the sun (considered as a black body) must be equal to the energy passing through the surface of a sphere whose radius is equal to the mean distance between the sun and the earth (= L), i.e.

$$(4\cdot\pi\cdot r^2)\cdot\sigma\cdot T_{sun}^{\ 4} = (4\cdot\pi\cdot L^2)\cdot G_s \qquad ...(13.96)$$

where, r = radius of the sun, and L = mean distance between the sun and earth. By this method, effective surface temperature of the sun is determined to be 5762 K.

Solar energy incident on earth's surface consists of two parts: **direct solar radiation,** G_D (which reaches the surface without any attenuation in the atmosphere) and **diffuse solar radiation** G_d (scattered radiation coming uniformly from all directions). Then, **total solar energy** incident on a horizontal surface is:

$$G_{solar} = G_D\cdot\cos(\theta) + G_d \text{ W/m}^2 \qquad ...(13.97)$$

where, θ is the angle between the sun's rays and the normal to the surface.

Constituents of the atmosphere absorb/scatter some of the solar radiation, as already mentioned; in addition, they also emit radiation. Main constituents contributing to this 'atmospheric radiation' are CO_2 and H_2O molecules. **Effective sky temperature,** T_{sky}, is calculated assuming the atmosphere to be a blackbody, i.e.

$$G_{sky} = \sigma\cdot T_{sky}^{\ 4} \text{ W/m}^2 \qquad ...(13.98)$$

Value of T_{sky} varies from 230 K to 285 K, depending on the atmospheric conditions.

Sky radiation absorbed by a surface is:

$$E_{sky\_absorbed} = \alpha\cdot G_{sky} = \alpha\cdot\sigma\cdot T_{sky} = \varepsilon\cdot\sigma\cdot T_{sky}^{\ 4} \text{ W/m}^2 \qquad ...(13.99)$$

For a surface at temperature T_s, exposed to both solar and atmospheric radiation, net rate of heat transfer to the surface is:

$$q_{net\_rad} = \Sigma E_{absorbed} - \Sigma E_{emitted}$$

i.e. $$q_{net\_rad} = (\alpha_s\cdot G_{solar} + \varepsilon\cdot\sigma\cdot T_{sky}^{\ 4}) - \varepsilon\cdot\sigma\cdot T_s^{\ 4}$$

i.e. $$q_{net\_rad} = \alpha_s\cdot G_{solar} + \varepsilon\cdot\sigma\cdot(T_{sky}^{\ 4} - T_s^{\ 4}) \text{ W/m}^2 \qquad ...(13.100)$$

Remember that incident solar energy coming from the sun originates at a very high temperature, and therefore, its spectral distribution is concentrated on short wavelength region; however, radiation emitted by the surface is from a relatively low temperature, and its spectral distribution is concentrated at infra-red region. This means that radiation properties (such as absorptivity and emissivity) for a surface are quite different for incident and emitted radiations. Table 13.8 lists values of solar absorptivity, α_s and emissivity ε (at 300 K) for some common materials. Obviously, solar collectors, widely used in solar energy applications, must be made of materials having high α_s and low ε.

TABLE 13.8 Solar absorptivity (α_s) and emissivity (ε) at room temperature for a few surface

Surface	α_s	ε
Aluminium		
Polished	0.09	0.03
Anodized	0.14	0.84
Foil	0.15	0.05
Copper		
Polished	0.18	0.03
Tarmished	0.65	0.75

Contd.

Contd.

Stainless steel		
Polished	0.37	0.60
Dull	0.50	0.21
Concrete	0.60	0.88
White marble	0.46	0.95
Red brick	0.63	0.93
Aspholt	0.90	0.90
Black paint	0.97	0.97
White paint	0.14	0.93
Snow	0.28	0.97
Human skin	0.62	0.97

13.13 Summary

Radiation heat transfer is unique as compared to other two modes of heat transfer, namely, conduction and convection, in the sense that no medium is required for radiation heat transfer to occur. Radiation involves electromagnetic waves of all wavelengths, ranging from zero to infinity. All bodies at temperatures above zero Kelvin emit radiation; our interest in this chapter has been on 'thermal radiation', i.e. radiations in the wavelength range of 0.1 to 100 microns.

After studying fundamental laws governing radiation heat transfer, we studied radiation properties of surfaces, such as absorptivity (α), emissivity (ε) and transmissivity (τ), since these properties affect the radiation heat transfer.

Radiation heat transfer between surfaces is also dependent on the relative size and orientation of the surfaces. This is taken care of in calculations by introducing the concept of 'view factor'. Analytical relations for view factor are available only for simple geometries, and mostly, graphical solutions, available in heat transfer hand-books, have to be referred to. Analytical relations and graphs for view factors for some of the commonly required geometries have been given. 'View factor algebra' enables one to get view factors for some complicated geometries, by 'breaking down' these geometries into simpler geometries for which values of view factors are either already known or tabulated.

Next, radiation heat transfer between surfaces in two-surface and three-surface enclosures were considered, using the radiation network method. This method greatly simplifies the analysis and gives a 'physical feel' of the problem. Important practical examples of two-surface enclosure are: two infinite, parallel planes, long concentric cylinders and concentric spheres. Furnaces with re-radiating (insulated) surfaces are examples of three-surface enclosure.

'Radiation shielding' to reduce the radiation heat transfer between surfaces was studied next. Importance of radiation shielding in reducing the radiation error in temperature measurement was studied.

Radiation has to be generally considered when the operating temperature level is high; as a rule, it will be prudent to check its relevance in problems involving natural convection and forced convection at high temperatures. Typical example is heat transfer from walls and doors of furnaces. In such problems, concept of 'radiation heat transfer coefficient' simplifies the numerical calculations.

Finally, after giving a brief introduction to radiation heat transfer from gases, vapours and flames, we made a mention of solar and atmospheric radiation, in view of its importance in the context of renewable energy sources.

Questions

1. What is meant by 'thermal radiation'? To which part of electromagnetic spectrum it belongs?
2. What is 'visible light'? To which part of electromagnetic spectrum it belongs?
3. A local radio station broadcasts radio waves at a wavelength of 480 m. What is the frequency of those radio waves?
4. Define: absorptivity, reflectivity and transmissivity. [M.U.]
5. Explain the following: (i) Black body and Grey body (ii) Specular reflector and Diffuse reflector (iii) Radiosity and Irradiation. [M.U.]
6. State Planck's law of monochromatic radiation. What is its significance? [M.U.]

7. State and explain Kirchhoff's law of radiation. [M.U.]
8. State Wein's law of displacement and prove that monochromatic emissive power of a black body is maximum when $\lambda_m . T = 2900\ \mu$mK. [M.U.]
9. What is intensity of radiation? Prove that total emissive power is π times the intensity of radiation. [M.U.]
10. Explain what is meant by 'Greenhouse effect'.
11. What is meant by 'view factor'? When is the view factor of a surface to itself equal to zero?
12. Write a short note on properties of view factor. [M.U.]
13. Explain 'crossed-strings method' of finding out view factors. When is it applicable?
14. Derive a general equation to find out the view factor of any cavity w.r.t. itself.
15. What is meant by 'view factor algebra'? When is it resorted to?
16. Write a short note on 'electrical network method' to determine radiant heat exchange between grey surfaces.
17. What is a 'radiation shield'? When is it used?
18. What is 'radiation error' in temperature measurement? Explain how radiation error can be reduced by the use of radiation shields.
19. How is radiation from a gas mass different from radiation from a solid?
20. What is 'mean path (or beam) length'?
21. How do you find out the emissivity of a gas mass containing carbon dioxide or/and water vapour, the mixture pressure being one atmosphere?
22. What is 'solar constant'? How is the effective surface temperature of sun determined when the value of solar constant is known?
23. What is meant by 'effective sky temperature'?
24. Why is solar absorptivity of a given surface quite different from its absorptivity for radiation from other surrounding bodies?

Problems

1. A hole of area $dA = 2$ cm^2 is opened on the surface of a large spherical cavity whose inside is maintained at 900 K. Calculate: (a) the radiation energy streaming through the hole in all directions into space, (b) the radiation energy streaming per unit solid angle in a direction making a 45 deg. angle with the normal to the surface of the opening.
2. The temperature of a body of area 0.1 m^2 is 700 K. Calculate the total rate of energy emission, intensity of normal radiation in W/(m^{2}sr), maximum monochromatic emissive power, and wavelength at which it occurs.
3. Treating sun as a black body with a surface temperature of 5800 K, determine the rate at which infra-red radiation ($\lambda = 0.76 - 100\ \mu$m) is emitted by the sun.
4. Filament of an incandescent light bulb is at 2800 K. Treating it as a black body, determine the fraction of the radiant energy emitted by the filament that falls in the visible range. Also, find out at what wavelength is the emission of radiation from the filament becomes maximum.
5. Window glass transmits radiant energy in the wavelength range 0.4 μm to 2.5 μm. Determine the rate of radiant energy which is transmitted, through a glass window of size: 2 m × 2 m, when the black body source temperature is: (a) 5800 K (i.e. sun's surface temperature), and (b) 1000 K .
6. Spectral emissivity of a particular surface at 900 K is approximated by a step function, as follows: $\varepsilon_1 = 0.3$ for $\lambda = 0$ to 2 μm, $\varepsilon_2 = 0.6$ for $\lambda = 2$ to 10 μm, and $\varepsilon_3 = 0.3$ for $\lambda = 10\ \mu$m to ∞. Calculate (i) average emissivity of the surface, and (ii) rate of radiation emission from the surface.
7. Two diffuse surfaces, a small disk of area A_1 and a large disk of area A_2, are parallel to each other and directly opposed, i.e. a line joining their centres is normal to both the surfaces. The large disk has a radius R and is located at height L from the smaller disk. Obtain an expression for the view factor of small disk w.r.t. the large disk. [M.U.]
8. Find out the net heat transferred between two circular disks 1 and 2, oriented one above the other, parallel to each other on the same centre line as shown in Fig. 13.18. Disk 1 has a radius of 0.6 m and is maintained at 900 K, and disk 2 has a radius of 0.7 m and is maintained at 600 K . Assume both the disks to be black surfaces.
9. Find out the net heat transferred between two aligned parallel rectangles, as shown in Fig. 13.18. ($X = 1$ m, $Y = 1.5$ m and $L = 1.5$ m). Surface 1 is maintained at 600 K, and surface 2 is maintained at 1000 K . Assume both the surfaces to be black surfaces.
10. Find out the net heat transferred between the areas A_2 and A_3 shown in Fig. Example 13.10 (See text for the figure). Area 1 is maintained at 700 K, and area 2 is maintained at 400 K. Assume both the surfaces to be black.
11. Determine the view factor from the side surface to the base of a cylindrical enclosure whose height is twice its diameter.

12. Determine the view factors from the base of a cube to each of the other five surfaces.
13. Find out the view factor from the dome of a hemispherical furnace to its circular base.
14. Find out the view factor (F_{ij}) between the plates i and j shown in Fig. Example 13.17a. Given: $w_i = 1$ m, $w_j = 2$ m and $L = 0.70$.
15. A 0.3 m × 0.3 m ingot, 1.2 m in height, at a temperature of 1000 deg.C, is taken out of a furnace and rests on the floor of a foundry room. Assuming that the surroundings are at a temperature of 30 deg.C, and the emissivity of the surface of the ingot to be 0.8, calculate the net radiant heat loss from the ingot.
16. A spherical liquid oxygen tank, 0.3 m in diameter is enclosed concentrically in a spherical container of 0.4 m diameter and the space in between is evacuated. The tank surface is at −183°C and has an emissivity = 0.2. The container surface is at 25°C and has an emissivity = 0.25. Determine the net radiant heat transfer rate. [M.U.]
17. A hemispherical furnace of radius 1.6 m has a roof temperature of $T_1 = 900$ K and emissivity 0.8. The flat circular floor has a temperature of 500 K and emissivity of 0.5. Calculate the net radiant heat exchange between the roof and floor. [M.U.]
18. Three thin-walled, long, circular cylinders 1, 2 and 3, of diameters 20 cm, 30 cm and 40 cm, respectively, are arranged concentrically. Temperature of cylinder 1 is 100 K and that of cylinder 3 is 300 K. Emissivities of cylinders 1, 2 and 3 are 0.05, 0.1 and 0.2, respectively. Assuming that there is vacuum inside the annular spaces, determine the steady state temperature attained by cylinder 2.
19. A long pipe, 50 mm diameter passes through a room and is exposed to air at 20°C. The pipe surface temperature is 93°C. Assuming that the emissivity of pipe surface is 0.6, calculate the radiation heat loss per metre length of the pipe. [M.U.]
20. Calculate the net radiant heat interchange per square metre for two large parallel plates maintained at 800°C and 300°C. The emissivities of two plates are 0.3 and 0.6, respectively.
21. Pipe carrying steam, OD = 20 cm, is exposed in a large room at 30°C. Pipe surface temperature = 400°C and emissivity of pipe surface is 0.8. Calculate heat loss by pipe by radiation. What would be rate of loss of heat if pipe is enclosed in a 40 cm diameter brick conduit of emissivity 0.91?
22. A blind cylindrical hole of diameter and length 3 cm is drilled into metal slab having emissivity 0.6. If the metal slab is maintained at temp 350°C, find the rate of heat escaping out of the hole by radiation. [M.U.]
23. Calculate the radiation heat transfer from a hemispherical cavity if inside temperature is 800 K and its emissivity is 0.6. Diameter of cavity is 500 mm.
24. A hohlraum is to be constructed out of a thin copper sphere of diameter = 20 cm. Its internal surface is highly oxidised. What should be the area of a small opening to be made on the surface of the sphere, if the desired absorptivity is 0.95?
25. A long duct of equilateral triangular section, of side $w = 1.0$ m, shown in Fig. Example 13.22, has its surface 1 at 600 K, surface 2 at 1100 K, and surface 3 is insulated. Further, surface 1 has an emissivity of 0.8 and surface 2 is black. Determine the rate at which energy must be supplied to surface 2 to maintain these operating conditions.
26. Two co-axial cylinders of 0.5 m and 1 m diameter are 1.2 m long. The annular top and bottom surfaces are well insulated and act as re-radiating surfaces. The inner surface is at 1100 K and has an emissivity of 0.6. The outer surface is maintained at 500 K and its emissivity is 0.4.
    (i) Determine the heat exchange between the surfaces
    (ii) If the annular base surfaces are open to the surroundings at 300 K, determine the radiant heat exchange.
    (Hint: If the outer cylinder is surface 2, first determine F_{21} and F_{22}).
27. Two parallel plates, 0.5 m × 1 m each, are spaced 0.5 m apart. The plates are at temperatures of 900°C and 600°C and their emissivities are 0.2 and 0.5, respectively. The plates are located in a large room, the walls of which are at 25°C. The surfaces of the plates facing each other only exchange heat by radiation. Determine the rates of heat lost by each plate and heat gain of the walls by radiation. Use radiation network for solution.
    Assume shape factor between parallel plates: $F_{12} = F_{21} = 0.285$.
28. A furnace is of the shape of a frustrum of a cone. Diameters of top and bottom surfaces are 5 m and 3 m, respectively, and the height is 3 m. Bottom surface is maintained at 1000°C and the top surface is at 600°C. Emissivities of top and bottom surfaces are 0.8 and 0.9, respectively. Inclined side surface is refractory surface. Find the radiation heat transfer from the bottom to the top surface and also the temperature of the inclined surface.
29. Two very large parallel plates with emissivities 0.2 and 0.6 exchange heat. Find the percentage reduction in heat transfer when two polished aluminium radiation shields ($\varepsilon = 0.3$) are placed between them. Also, find the equilibrium temperatures of the two shields.
30. Two large parallel planes facing each other and having emissivities 0.3 and 0.5 are maintained at 700°C and 500°C, respectively. Determine the rate at which heat is exchanged between the two surfaces by radiation. If a radiation shield of emissivity 0.05 on both sides is placed parallel between the two surfaces, determine the percentage reduction in the radiant heat exchange rate. What is the equilibrium temperature of the shield?

31. A spherical tank with diameter D_1 = 30 cm filled with a cryogenic fluid at T_1 = 90 K is placed inside a spherical container of diameter D_2 = 50 cm and is maintained at T_2 = 300 K. Emissivities of inner and outer tanks are ε_1 = 0.10 and ε_2 = 0.2, respectively. A spherical radiation shield of diameter D_3 = 40 cm and having an emissivity ε_3 = 0.05 on both surfaces is placed between the spheres. Calculate the rate of heat loss from the system by radiation. Then, find the rate of evaporation of cryogenic liquid for $h_{fg} = 2.1 \times 10^5$ J/kg. What is the equilibrium temperature of the shield?

32. A double-walled flask may be considered as equivalent to two infinite parallel plates. The emissivities of walls are 0.3 and 0.8, respectively. The space between the walls of the flask is evacuated. To reduce heat flow, a shield of polished aluminium with emissivity equal to 0.04 (on both sides) is inserted between the two walls. Find the percentage reduction in heat transfer. Also, find the equilibrium temperature of the shield. [M.U.]

33. The net radiation from the surface of two parallel plates maintained at temperatures T_1 and T_2 is to be reduced to one-fifth. Calculate the number of screens to be placed between two surfaces to achieve this reduction in heat exchange, assuming the emissivity of screens on both sides as 0.05 and that of surfaces as 0.2.

34. A 10 mm OD pipe carries a cryogenic fluid at 100 K. This pipe is encased by another pipe of 15 mm OD, and the space between the pipes is evacuated. The outer pipe is at 300 K. Emissivities of inner and outer surfaces are 0.1 and 0.2, respectively. (a) Determine the radiant heat flow rate over a pipe length of 3 m. (b) If a radiation shield of diameter 12 mm and emissivity 0.05 on both sides is placed between the pipes, determine the percentage reduction in heat flow. (c) What is the equilibrium temperature of the shield?

35. Hot air is flowing in a duct whose walls are maintained at a temperature T_w = 500 K. A thermocouple placed in the stream shows a reading of 800 K. If the emissivity of the thermocouple junction is ε_c = 0.8 and the convective heat transfer coefficient between the flowing air and the thermocouple is h = 80 W/(m^{2}C), find out the true temperature of the flowing stream. How much is the radiation error?

36. Hot air is flowing in a duct whose walls are maintained at a temperature T_w = 400 K. A thermocouple placed in the stream shows a reading of 600 K. If the emissivity of the ther-mocouple junction is ε_c = 0.6 and the convective heat transfer coefficient between the flowing air and the thermocouple is h = 100 W/(m^{2}C), find out the true temperature of the flowing stream. (b) Now, if a radiation shield (ε_s = 0.2) is placed between the thermocouple and the walls, what will be new value of T_c read by the thermocouple? And, how much is the temperature error? Take A_c/A_s = 0.1.

37. A spherical chamber of 1.5 m diameter is filled with a gas mixture at 1 atm. and is at 1200 K. The gas mixture contains 18% CO_2 by volume, and the rest of the mixture is non-radiating gases. Determine the emissivity of the gas body.

(b) If the volume is filled to a pressure of 3 atm., but with the fraction of CO_2 still being 18 %, what will be the value of emissivity of gas body?

38. A cubical furnace of 2 m side, contains a gas mixture at 1500 K at a total pressure of 2 atm. The gas mixture contains 15% of CO_2 and 10% of H_2O by volume. If the furnace walls are at a temperature of 600 K, find out the heat transferred by radiation from the gases to the walls. Assume that surfaces are black.